The electron and the holographic mass solution[a)]

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Abstract: A computation of the electron mass is found utilizing a generalized holographic approach in terms of quantum electromagnetic vacuum fluctuations. The solution gives a clear insight into the structure of the hydrogen Bohr atom, in terms of the electron cloud and its relationship to the proton and the Planck scale vacuum fluctuations. Our electron mass solution is in agreement with the measured Committee on Data of the International Council for Science (CODATA) 2014 value. As a result, an elucidation of the source of the fine structure constant, the Rydberg constant, and the proton-to-electron mass ratio is determined to be in terms of vacuum energy interacting at the Planck scale. © *2019 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-32.2.255]

Résumé: La masse de l'électron est calculée par l'utilisation d'une approche holographique généralisée considérant les fluctuations du vide quantique électromagnétique. La solution obtenue donne une image claire de la structure de l'atome d'hydrogène de Bohr constitué d'un nuage électronique lié au proton en lien avec les fluctuations du vide à l'échelle de Planck. Notre solution pour la masse de l'électron est en accord avec la valeur CODATA (Comité des Données du Conseil International pour la Science) 2014. Grâce à ce résultat, nous avons pu établir l'origine de la constante de structure fine, de la constante de Rydberg et du rapport masse protons/électrons en fonction de l'énergie du vide à l'échelle de Planck.

Key words: Electron; Hadron Mass; Holographic; Quantum Gravity; Entropy.

I. INTRODUCTION

The electron mass is typically determined utilizing penning traps, where measurements of the cyclotron frequency for both an electron and a reference ion can be made. The latest measured value given by the Committee on Data of the International Council for Science (CODATA) is $9.10938356(11) \times 10^{-28} g$ with a relative uncertainty of 1.2×10^{-8}.[1] More recent indirect methods combine Penning trap measurements of the Larmor-to-cyclotron frequency ratio with a corresponding very accurate electron spin g-factor calculation and find the more precise values of $0.000548579909067(14)(9)(2)$ u $(9.109389919 \times 10^{-28} g)$ and $0.000\,548\,579\,909\,065(16)$ u $(9.109389919 \times 10^{-28} g)$, respectively, with a relative uncertainty of order 10^{-11}.[2,3]

These measurements are extremely precise, and yet a satisfactory derivation from first principles remains to be found, and thus, the nature of the electron remains a mystery.

The standard definition for the mass of the electron is therefore generally given in terms of the Rydberg constant R_∞,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = 9.10938356(11) \times 10^{-28} g, \tag{1}$$

where h is Planck's constant and α is the fine structure constant.

However, although in agreement with the measured CODATA 2014 value, this standard form does not reveal the nature or structure of the electron. As noted by Wilczek, *"An electron's structure is revealed only when one supplies enough energy [...] at least 1 MeV, which corresponds to the unearthly temperature of 10^{10} kelvin"* below which it "appears" pointlike and structureless.[4]

Although the position and momentum can only be defined in terms of a probability cloud, the quantum behavior of the electron is successfully calculated by the current standard model. Yet the most precise prediction, being that of the g-factor,[5,6] still requires the inclusion of a contribution from quantum vacuum fluctuations[7] to account for the observed deviation known as the anomalous magnetic moment.[8]

Quantum corrections are also expected for an electric field—but as yet no such field has been detected. Based on charge-parity (CP) violating components, the standard model assumes an upper limit on the electron electric dipole moment (EDM) of $d_e \leq 10^{-38} q$ cm,[9] which is smaller than current experimental sensitivities. However, recent experiments confirm a nonzero EDM with a much higher upper limit, e.g., Refs. 10 and 11, and more recently Ref. 12 who find $d_e < 10.5 \times 10^{-28} q$ cm, $d_e < 6.05 \times 10^{-25} q$ cm and $d_e < 1.1 \times 10^{-29} q$ cm, respectively, suggesting the standard model is incomplete and there must be other sources of CP violation. Higher EDMs are predicted by extensions to the standard model, e.g., supersymmetric models, which predict $d_e > 10^{-26} q$ cm (Ref. 13) and are in agreement with the results from Ref. 11 but not Refs. 10 and 12. In either case, it is clear that current models, the standard model and extensions such as supersymmetric models, are incomplete.

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Defining the fundamental characteristics of particles from first principles, and without free parameters, is of great importance as not only will it provide information about the structure of subatomic particles but also the source of mass and the nature of spacetime itself. Successful predictions allow us to confirm and improve upon existing models.

In the standard approach, quantum chromodynamics (QCD), hadron masses are determined by considering not only the quark masses but also and most importantly the dynamics of the system. Due to the nonlinear nature of the strong force, exact calculations of nucleons and their constituent parts are extremely difficult and thus rely on numerical techniques where probability amplitudes are assigned to each Feynman diagram and Monte Carlo simulations (or other similar iterative methods) determine the best fit. However, despite the development of sophisticated numerical techniques and ever faster super computers, QCD calculations have been unable to successfully predict the mass of the proton.

In an effort to make a reasonable prediction, Durr *et al.*[14] utilized a computational technique called lattice gauge theory. In the lower energy regime (i.e., lower than proton energy) where the interactions are strong, and the coupling parameter is large,[15] a nonperturbative approach is required where a discrete set of spacetime points rather than a spacetime continuum allows for improved calculations. In the model used by Durr *et al.*,[14] only three input parameters were required: the light (up and down) quark mass; the strange quark mass; and the gauge coupling parameter, g. These calculations cannot distinguish between a proton or a neutron and thus yield a general value for a nucleon of $m_N = 936\,\mathrm{MeV}/c^2 \pm 25/\pm 22 = 1.67\times 10^{-24} \pm\ 0.0446g$.[1] This value is in good agreement with the general mass of a nucleon but, based on the time-intensive methods, is not yet as good as expected. Furthermore, there is no analytical solution to lattice QCD (LQCD) or a good understanding of the nature of confinement. There is no doubt that QCD is successful at calculating these measured parameters. However, as noted by Wilczek,[16] there are limitations to the purely mathematical approach of QCD, and he thus suggests, along with the asymptotic method, a more simplistic approach which looks at the underlying physical model.

Starting with the premise that an electron cloud can be considered as an "electron" coherent field of information, we look at the microstructure of the electron system from a generalized holographic approach. In Section II, we will give an overview of this generalized holographic approach, which in previous work successfully computes the mass of the proton[17] and a precise charge radius of the proton within an 1σ agreement with the latest muonic measurements,[18] relative to a 7σ variance in the standard approach.[19]

Utilizing this approach, we find an electron mass solution in terms of the surface-to-volume entropy measured as Planck oscillator information bits. This value is in agreement with the measured CODATA 2014 value.

II. THE HOLOGRAPHIC PRINCIPLE AND THE PROTON MASS

In previous works,[17,18,20] a quantized solution to gravity is given in terms of Planck Spherical Units (PSUs) in a generalized holographic approach. This section gives a brief overview of the history and development of the holographic principle that led to the generalization offered in Refs. 17, 18, and 20.

The holographic principle has its origins in the work of Bohm[21,22] who suggested that every region contains a total "structure" enfolded within it. Bohm equated this idea with the Universe, which he referred to as a hologram based on its analogy with optical holography. Bekenstein later proposed, similarly, that entropy in a given region of space is limited by the area of its boundary.

Bekenstein's idea began as a solution to the violation of the second law of thermodynamics for black hole physics, which suggests that a black hole has entropy. When the second law of thermodynamics is considered in terms of black hole physics, the entropy of the black hole exterior is found to decrease. To remove such a violation, Bekenstein[23–25] proposed that the entropy S or information contained in a given region of space, such as a black hole, is proportional to its surface horizon area A expressed in units of Planck area

$$S \propto \frac{A}{\ell^2} \tag{2}$$

and obeys a generalized second law in which the black hole entropy plus the common entropy of the black-hole exterior never decreases. This relation between black hole physics and thermodynamics was also made between the first law of black hole mechanics and the first law of thermodynamics. The first law of black hole mechanics

$$dM = \frac{\kappa}{8\pi G}dA + \Omega dJ + \Phi dQ \tag{3}$$

gives the mass M in terms of the surface gravity κ, the surface area A, the angular velocity Ω, the angular momentum J, the electrostatic potential Φ, and the electric charge Q. (Note that for a Schwarzschild black hole, the angular momentum and electric charge are set to be zero.)

Whereas the first law of thermodynamics

$$dE = TdS - PdV \tag{4}$$

equates the energy in terms of the temperature T, the entropy S, the pressure P, and the volume V.

The quantities, A and κ of the black hole, have a close analogy with entropy S and temperature T, respectively, thus by equating the first terms on the right-hand side of each equation [Eqs. (3) and (4)], Bardeen *et al.*[26] were able to show that

$$\frac{\kappa}{8\pi G}dA = TdS \rightarrow S = \frac{\kappa A}{8\pi GT}. \tag{5}$$

In 1974, Hawking[27,28] predicted the spontaneous emission of black hole thermal radiation, arising from steady conversion of quantum vacuum fluctuations into pairs of particles, with a temperature

$$T_H = \frac{\hbar}{2\pi k_B \tau_\kappa}, \tag{6}$$

where k_B is the Boltzmann constant and τ_κ is the characteristic lifetime for the light pulse, emitted by the in falling matter, redshifted to zero (and is given as the time it takes light to travel a distance $2r_S$, where r_S is the Schwarzschild radius).

Substituting the above definition for Hawking temperature [Eq. (6)] and including a factor c^2/k_B, the entropy can be given in dimensionless units as

$$S = \frac{\kappa A 2\pi k_B c}{8\pi G\hbar\kappa}\frac{c^2}{k_B} = \frac{Ac^3}{4G\hbar} = \frac{A}{4\ell^2}. \tag{7}$$

The Bekenstein-Hawking entropy of a black hole expressed in units of Planck area is thus given as

$$S = \frac{A}{4\ell^2}, \tag{8}$$

where the Planck area, ℓ^2, is taken as one unit of entropy and A is the surface area of the black hole.

Bekenstein[29] further argued for the existence of a universal upper bound for the entropy of an arbitrary system with maximal radius r,

$$S \leq \frac{2\pi r E}{\hbar c} \tag{9}$$

and found that this maximal bound is equivalent to the Bekenstein-Hawking entropy for a black hole (assuming $E = mc^2$).

This idea of a maximal entropy, defined by the Bekenstein bound, along with unitarity arguments, eventually led to a holographic principle as described by 't Hooft[30–32] and later further developed by Susskind.[33] Through studying the quantum mechanical features of black holes and the third law of thermodynamics relating entropy to the total number of degrees of freedom, 't Hooft showed that the entropy directly counts the number of Boolean degrees of freedom and concluded that the relevant degrees of freedom of a black hole must not exceed $1/4$ the total surface area, and thus, the maximal entropy for a black hole is $A/4$. That is "a region with boundary of area A is fully described by no more than $A/4$ degrees of freedom, or about 1 bit of information per Planck area."

However, as noted by Bousso[32] for all systems larger than the Planck scale, the volume will exceed the surface area (e.g., for a proton, the volume is larger than the area by a factor of 10^{20}). Thus, the result obtained when only the surface is considered, is at odds with the much larger number of degrees of freedom estimated from the local field theory. The question thus arises whether the Bekenstein-Hawking entropy counts all states inside a black hole or only the states distinguishable to the exterior observer. This question is put into focus when considering "Wheelers bags of gold" which refers to classical solutions to Einstein's equations that allow values of the black hole entropy larger than those allowed by an area law.[34,35]

The nature of holography, the holographic principle, and the maximal entropy of a black hole is thus further explored by Haramein who proposes a generalized holographic approach in terms of both the surface and volume entropy of a spherical system.[17,18] We will give a brief outline of this approach here.

To begin with, it is important to note that the holographic bit of information is not defined as ℓ^2, as previously suggested,[23,25,30,31,33,36] and is instead defined as an oscillating PSU, given as

$$PSU = \frac{4}{3}\pi r_\ell^3, \tag{10}$$

where $r_\ell = \ell/2$ and ℓ is the Planck length.

These PSUs, or Planck voxels, tile along the area of a spherical surface horizon, producing a holographic relationship with the interior information mass-energy density (see Fig. 1).

In this generalized holographic approach, it is therefore suggested that the information/entropy of a spherical surface horizon should be calculated in spherical bits and thus defines the surface information/entropy in terms of PSUs, such that,

$$\eta = \frac{A}{\pi r_\ell^2}, \tag{11}$$

where the Planck area, taken as one unit of information/entropy, is the equatorial disk of a PSU, πr_ℓ^2 and A is the surface area of a spherical system. We note that in this definition, the entropy is slightly greater ($\sim$5 times) than that set by the Bekenstein Bound [Eq. (9)], and the proportionality constant is taken to be unity (instead of $1/4$ as in the Bekenstein-Hawking entropy). It has been previously suggested that the quantum entropy of a black hole may not exactly equal $A/4$.[35,37] To differentiate between the models,

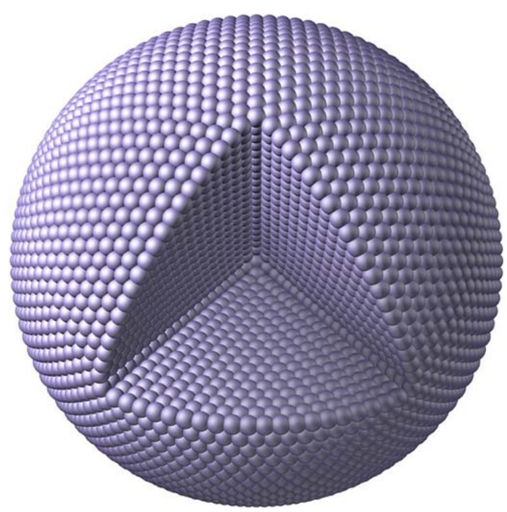

FIG. 1. (Color online) Schematic to illustrate the PSUs packed within a spherical volume.

the information/entropy S encoded on the surface boundary in Haramein's model is termed $\eta \equiv S$.

Following this definition for surface information η, the information/entropy within a volume of space is similarly defined in terms of PSUs as

$$R = \frac{V}{\frac{4}{3}\pi r_\ell^3} = \frac{r^3}{r_\ell^3}, \tag{12}$$

where V is the volume of the spherical entity and r is its radius.

In previous work,[17,18] following the relationship between mass and surface area [Eq. (3)] with energy and entropy [Eq. (4)], it was demonstrated that the holographic relationship between the transfer energy potential of the surface information and the volume information equates to the gravitational mass of the system. It was thus found that for any black hole of Schwarzschild radius r_S, the mass m_S can be given as

$$m_S = \frac{R}{\eta} m_\ell, \tag{13}$$

where m_ℓ is the Planck mass, η is the number of PSUs on the spherical surface horizon, and R is the number of PSUs within the spherical volume. Hence, a holographic gravitational mass equivalence to the Schwarzschild solution is obtained in terms of a discrete granular structure of spacetime at the Planck scale. It should be noted that this view of the interior structure of the black hole in terms of PSUs, is supported by the concept of black hole molecules and their relevant number densities as proposed by Miao and Xu[38] and Wei and Lui.[39] As well, the relationship between the interior structure in terms of "voxels" and the connecting horizon pixels is discussed in the work of Nicolini.[40]

Furthermore, this inequality in energy potential between the surface information and the volume information, where $R > \eta$ for all $r > 2\ell$ suggests that both, the gravitational curvature potential is the result of an asymmetry in the information structure of spacetime, and the volume information is not only the result of the information/entropy surface bound of the local environment but may also be nonlocal, due to wormhole interactions as those proposed by the ER = EPR conjecture, where black hole interiors are connected through microwormhole interactions.[41]

Moreover, we find that the only radius at which the holographic ratio equals one (i.e., $R = \eta$), where all the volume information is encoded on the surface, is the condition

$$r_{S_\ell} = \frac{2Gm_\ell}{c^2} = 2\ell, \tag{14}$$

where r_{S_ℓ} is the Schwarzschild radius of a black hole with mass $m = m_\ell$.

In this case, the surface entropy η and the volume entropy R are thus calculated to be

$$\eta_\ell = \frac{4\pi r_{S_\ell}^2}{\pi r_\ell^2} = \frac{4\pi(2\ell)^2}{\pi(\ell/2)^2} = 64, \tag{15}$$

$$R_\ell = \frac{r_{S_\ell}^3}{r_\ell^3} = \frac{(2\ell)^3}{(\ell/2)^3} = 64. \tag{16}$$

This results in a holographic ratio of $R_\ell/\eta_\ell = 1$ yielding $m_\ell = (R_\ell/\eta_\ell)m_\ell$ such that a balanced state of equilibrium between the volume-to-surface information transfer potential is achieved, supporting the conjecture that due to its ultimate stability, the Planck entity is the fundamental granular kernel structure of spacetime forming a crystal-like structured lattice at the very fine scale of the quantum vacuum.[42,43]

Additionally, it is important to note that there is a factor of 2 or $1/2$ between the Planck length and the Schwarzschild radius of a Planck mass black hole, and although its physical meaning has yet to be completely understood, it has been related to geometric considerations of motion, particle physics and cosmology, and commonly occurs in the most fundamental equations of physics.[44] However, the origin of this factor may be the result of the holographic surface-to-volume consideration of the fundamental geometric clustering of the structure of spacetime at the Planck scale, where one Planck mini black hole is a cluster bundle of Planck spherical vacuum oscillators.

Of course, these considerations on the granular structure of space lead to the exploration of the clustering of the structure of spacetime at the nucleonic scale, where it was found that a precise value for the mass m_p and charge radius r_p of a proton can be given as

$$m_p = 2\frac{\eta}{R} m_\ell = 2\phi m_\ell, \tag{17}$$

$$r_p = 4\ell \frac{m_\ell}{m_p} = 0.841236(28) \times 10^{-13}\,\text{cm}, \tag{18}$$

where ϕ is defined as a fundamental holographic ratio. Significantly, this value is within an 1σ agreement with the latest muonic measurements of the charge radius of the proton,[17,18] relative to a 7σ variance in the standard approach.[19]

The radius of an oscillating electrostatic field such as a proton defines an effective charge boundary in that region of space—a "charge radius." The standard approach thus relies on indirect measurements of the energy interaction at the charge surface boundary between the electron and the proton when measuring the Lamb shift quantum vacuum oscillations.[45–49] This is typically done utilizing electron proton scattering and/or hydrogen spectroscopy methods. Both these methods have consistently yielded similar results, where the latest 2014 CODATA value, $r_p = 0.8751(61) \times 10^{-13}$ cm, is based on a least-squares approximation between both methods.

III. DETERMINING THE MASS OF THE ELECTRON

In Section II, we described a generalized holographic approach which derives the proton mass from the granular Planck scale structure of spacetime in terms of a surface-to-volume information transfer potential.

The question is can this approach be extended to the electron? The first step in answering this question is to consider the spatial extent of the electron and the volume of information that it encloses. However, the spatial extent of the electron has not been conclusively defined. As we described in Section II, the generalized holographic approach sees the mass as emerging from the granular Planck scale structure of spacetime in terms of a surface-to-volume information transfer potential ϕ, which decreases with the increasing radius. Similarly, instead of thinking about the electron as a separate system, the electron could be thought of as a cloud of potential energy spatially extending from the proton out to the radius where the volume encloses the electron cloud of a hydrogen Bohr atom. Thus, in an attempt to deepen our understanding, we consider the holographic ratio relationship as we extend the radius of the comoving Planck particles to $r > r_p$.

Equation (17) therefore becomes

$$m_r = \beta \phi_r m_\ell, \tag{19}$$

where m_r is the mass of any spherical system with radius r, β is a geometric parameter, and ϕ_r is the holographic surface-to-volume ratio in terms of PSUs for any spherical system with radius r.

With $\beta = 1/2\alpha$ (refer to Section II on the factor of 2 in physics), we find a mass in precise agreement with the experimental mass of the electron when the holographic ratio reaches $r = a_0$, where a_0 is the Bohr radius.

The solution for the mass of the electron can thus be given as

$$m_e = \frac{1}{2\alpha}\phi_e m_\ell, \tag{20}$$

where

$$\phi_e = \frac{\eta_e}{R_e}, \quad \eta_e = \frac{4\pi a_0^2}{\pi r_\ell^2} \quad \text{and} \quad R_e = \frac{4/3\pi a_0^3}{4/3\pi r_\ell^3} = \frac{a_0^3}{r_\ell^3}.$$

With this solution, we find a mass of $m_e = 9.10938(30) \times 10^{-28} g$ which compared to the measured CODATA 2014 value is accurate within 1σ and with a precision of 10^{-5}.[1] The precision, and thus accuracy, of our solution is restricted by the value of the Planck units which are dependent on experimental values given for the gravitational constant, G. However, when the absolute value for the holographic mass solution for the electron is considered, the mass is comparable with the experimental CODATA 2014 value to a greater degree of accuracy $<1\sigma$ and a precision of 10^{-8} with a confidence level of $99.9\dot{9}\%$.

The presence of α in Eq. (20) reveals that both the charge and velocity are important contributing factors to the mass solution where, for the case of the electron at least, the holographic mass solution can be formulated in terms of both velocity and charge relationships

$$m_e = \frac{1}{2\alpha}\phi_e m_\ell = \frac{1}{2}\frac{v_\ell}{v_e}\phi_e m_\ell = \frac{1}{2}\frac{q_\ell^2}{q_e^2}\phi_e m_\ell, \tag{21}$$

where $\alpha = v_\ell/v_e$ and $\alpha = q^2/q_\ell^2$.

This solution, as well as being significantly accurate, gives us insight into the physical and mechanical dynamics of the granular Planck scale vacuum structure of spacetime and its role in the source of angular momentum, mass, and charge. The definition clearly demonstrates that the differential angular velocities of the collective coherent behavior of Planck information bits (PSUs) determines specific scale boundary conditions and mass-energy relationships, analogous to the collective behavior of particles in a rotating fluid[50] or superfluid plasma.[51]

This solution as well resolves the difficulty associated with hierarchy problems (we will address the electron-to-proton mass ratio below). The current quantum understanding resolves the hierarchy bare mass problem for the electron mass through the consideration of antimatter where positron and electron pairs pop in and out of the vacuum. These virtual particles smear out the charge over a greater radius such that the bare mass energy is canceled by the electrostatic potential, where the greater the radius the lesser the need for fine tuning. In the solution presented here, the electron is extended to a maximal radius of a_0 and we are able to demonstrate that the mass of the electron is a function of the Planck vacuum oscillators surface-to-volume holographic relationship, over this region of spacetime. The hierarchy bare mass problem is thus resolved by considering Planck vacuum oscillators acting coherently extending over a region of space equivalent to the Bohr hydrogen atom.

In much the same way that the electron analogy is proposed to resolve the Higgs hierarchy problem, with the inclusion of virtual supersymmetric particles, we could also assume that the surface-to-volume holographic relationship in the Higgs region of space would solve for the mass of the Higgs, where the Higgs radius would be of the order, $r_\ell < r_{Higgs} < r_p$.

The hierarchy problem associated with the mass of the electron and the mass of the proton can also be understood in terms of the surface-to-volume holographic ratio over their respective commoving regions of space, where the greater the radius the smaller the mass. The mass is thus a direct function of the commoving behavior of the Planck vacuum, where the spin and mass decrease as a function of the increasing radius.

IV. EXTENSION OF THE HOLOGRAPHIC SOLUTION FOR RADII LESS THAN THE BOHR RADIUS

When we further extend this solution for the n = 1 state, we find that at radii $r < a_0$, the holographic mass solution increases as shown in Fig. 2. This application of the holographic solution gives us the following equation

$$\frac{1}{2\alpha}\phi_r(r) m_\ell = N m_e, \tag{22}$$

where $\phi_r(r)$ is the holographic ratio as a function of the radius r for $r < a_0$, and N is an integer.

With this solution, we could recognize N as being the atomic number Z, where for progressively smaller fractions of a_0, we find an interesting proportional relationship between the holographic mass and the mass of the electron (see Fig. 2).

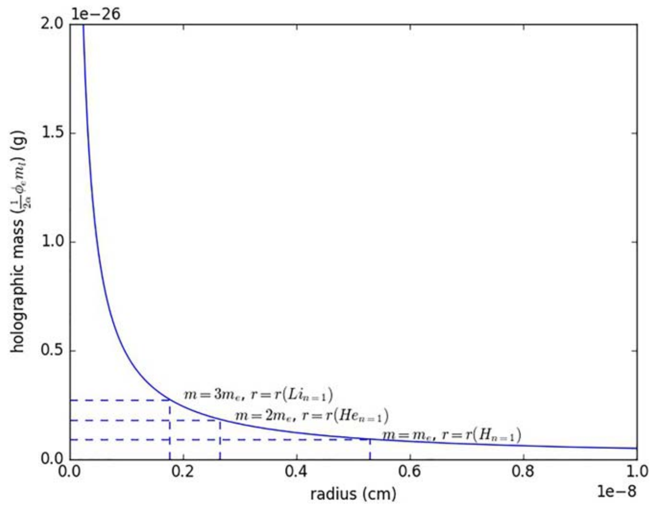

FIG. 2. (Color online) Graph to show the holographic mass solution as a function of radius. Note that the holographic mass is equal to m_e at the corresponding radii of a_0/N. For example, the holographic mass: equals the mass of one electron at a radius of the hydrogen atom in its n = 1 state; equals the mass of two electrons at a radius of the helium atom in its n = 1 state; equals the mass of three electrons at a radius of the lithium atom in its n = 1 state, and so on. Note that this relationship is only shown on the graph for the first three elements but continues for all known elements.

From this holographic mass solution, we are thus able to calculate the total mass of the electrons for all known elements, without the need for adding the atomic mass number, Z. We instead find that the atomic mass number Z could be a natural consequence of the holographic solution. As a result, a picture develops in which the structure of the Bohr atom and the charge and mass of both the proton and the electron are consequences of spin dynamics in the comoving behavior of the Planck scale granular structure of spacetime. This suggests that the confinement for the electron is a result of the quantum gravitational force exerted by the dynamics of the vacuum at the Planck scale. The electrostatic force can thus be accounted for in the same way the strong force is accounted for in the case of the proton,[17,18] where in both cases, the proton and the electron confinement are the result of a quantum force exerted through the granular Planck scale structure of spacetime.

V. DERIVING THE RYDBERG CONSTANT, THE FINE STRUCTURE CONSTANT, AND THE PROTON TO ELECTRON MASS RATIO

A. The Rydberg constant

The Rydberg constant is considered to be one of the most well-determined physical constants, with an accuracy of 7 parts to 10^{12} and is thus used to constrain the other physical constants.[1,6] However, as the same spectroscopic experiments are used to determine both the charge radius of the proton and the Rydberg constant, the recent muonic measurements of the charge radius of the proton implies that the Rydberg constant would change by $4-5\sigma$.[52–54] This is known as the proton radius puzzle. The standard formula for the Rydberg constant is given as

$$R_\infty = \frac{m_e \alpha^2 c}{2h}, \tag{23}$$

so any change in the experimentally determined value for the proton radius and thus the Rydberg constant will have a significant effect on the constraints defining the relationships between m_e, α, c, and h.

In order to understand and subsequently infer any discrepancies between experimental and theoretical values, it is important to determine the underlying physical mechanism under which the Rydberg constant R_∞ emerges. The holographic mass solution offers such a geometric mechanism providing a physical description and insight into how R_∞ emerges.

The standard formula for the mass of the electron [Eq. (1)] can be reduced to

$$m_e = \frac{2R_\infty h}{c\alpha^2} = \frac{4\pi\ell m_\ell R_\infty}{\alpha^2}. \tag{24}$$

Equating this, Eq. (24), with the geometric solution, Eq. (20) gives

$$m_e = \frac{4\pi\ell m_\ell R_\infty}{\alpha^2} = \frac{1}{2\alpha}\phi_e m_\ell,$$

and thus,

$$R_\infty = \frac{\alpha\phi_e}{8\pi\ell} = 1.097373(36)\times 10^5\ \mathrm{cm}^{-1}. \tag{25}$$

This definition offers a geometric solution for the Rydberg constant in agreement with the experimentally determined CODATA 2014 value.

As shown above, the holographic mass solution yields a correct value for both the charge radius of the proton, in agreement with the muonic radius measurement[17,18] and the Rydberg constant independently, resolving the proton radius puzzle from first principles.

B. The fine structure constant and the proton-to-electron mass ratio

This approach can as well be extended to derive the fine structure constant, α and the proton to electron mass ratio, μ, in terms of ϕ_e.

If we equate the new geometric solution, Eq. (25), with the standard definition, Eq. (23), we get

$$R_\infty = \frac{m_e\alpha^2 c}{2h} = \frac{\alpha\phi_e}{8\pi\ell},$$

and thus,

$$\alpha = \frac{\phi_e h}{8\pi r_\ell m_e c} = \frac{\phi_e \lambda_e}{8\pi r_\ell} = 7.29735(34)\times 10^{-3}, \tag{26}$$

which is in agreement with that of the CODATA 2014 value. The ratio of the proton mass to the electron mass, μ, can also be given in terms of the geometric solution, Eqs. (17) and (20),

$$\mu = \frac{m_p}{m_e} = \frac{2\phi m_\ell}{\phi_e m_\ell / 2\alpha} = 4\alpha \frac{\phi}{\phi_e} = 4\alpha \frac{a_0}{r_p} = 1836.152(86). \tag{27}$$

A similar relationship, $m_e/m_p \sim 10\alpha^2$, was identified by Carr and Rees in 1979 [Eq. (45) of Ref. 55] which they state is from a coincidence in nuclear physics and note that if the relation was not satisfied, elements vital to life would not exist.[55]

VI. SUMMARY

A new derivation for the mass of the electron is presented from first principles, where the mass is defined in terms of the holographic surface-to-volume ratio and the relationship of the electric charge at the Planck scale to that at the electron scale. It should be emphasized that the generalized holographic approach is a new approach to quantum gravity based on geometrical considerations alone. It therefore does not utilize the established mathematics of quantum mechanics and general relativity to achieve its objectives. Interestingly and nontrivially, it is able to give a quantum analogy for the mass of the black hole—in that its mass is determined from discrete voxels of spacetime—and as well extends to the nucleon scale, where the mass of the proton and the electron can similarly be defined.

This new derivation for the electron extends the holographic mass solution to the hydrogen Bohr atom and for all known elements, defining the atomic structure and charge as a consequence of the electromagnetic fluctuation of the Planck scale. Furthermore, the atomic number, Z, emerges as a natural consequence of this geometric approach. The confinement for both the proton and the electron repulsive electrostatic force are now accounted for by a quantum gravitational force exerted by the granular Planck scale structure of spacetime. We conclude that this new approach offers an accurate value for the mass of the electron. As well, contrary to the standard calculation [as shown in Eq. (1)], it offers a physical understanding to the structure of spacetime at the quantum scale, yielding significant insights into the formation and source of the material world. Our results support our belief that such insights have significant value and should be developed further.

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