The *zitterbewegung* electron puzzle

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Abstract: This work is an updated revision of semiclassical descriptions for the electron, including the fully relativistic QED-P model from H. J. Wilson based on the original Dirac equation (DE). The models presented hereafter go beyond the depiction of the electron as a structureless nondimensional point like charge with momentum and position determined by a probabilistic interpretation of the wavefunction described in terms of an electronic density cloud. These models share features in common that provide useful insights concerning the nature of the electron; for instance, they all consider *zitterbewegung*, a light speed "trembling-along-the-way" electron motion, to be a real oscillatory motion of the electron. The last model presented in this review is the electron mass model from Val Baker *et al*. [Phys. Essays **32**, 255 (2019)], where the electron mass is defined in terms of a holographic surface-to-volume ratio ϕ and the relationship of the electric charge at the Planck scale to that at the electron scale, obtaining a value in agreement with the latest CODATA value. We discuss the relationship between these models. The large number of correspondences between the models should not be taken lightly and indicate, in our view, that something very fundamental about the nature of the electron is being put forward by this study.
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Résumé: Ce travail est une revue des modèles semi-classiques de l'électron, avec notamment le modèle électrodynamique QED-P de H. J. Wilson entièrement relativiste, basé sur l'équation de Dirac (DE). Les modèles présentés vont au-delà de la représentation de l'électron comme une charge ponctuelle sans dimension ni structure. La quantité de mouvement et la position de l'éctron sont déterminées par une interprétation probabiliste de la fonction d'onde décrite comme un nuage de densité électronique. Ces modèles semi-classiques possèdent des caractéristiques comunes qui nous permettent de mieux décrire la nature de l'électron; par exemple, ils considèrent tous que le *zitterbewegung*, un 'mouvement de tremblement' de l'électron à la vitesse de la lumière, est un mouvement oscillatoire réel de l'électron. Le dernier modèle de cette revue est le modèle de la masse holographique de l'électron, développé par A. Val Baker, O. Alirol et N. Haramein [Phys. Essays **32**, 255 (2019)], où la masse de l'électron est définie en fonction du rapport holographique surface-volume ϕ et du ratio entre la charge électrique à l'échelle de Planck et celle à l'échelle de l'électron, ce qui permet d'obtenir une valeur de la masse de l'électron en accord avec la valeur la plus récente de CODATA. Nous discutons également de la relation entre ces modèles. Le nombre important de correspondances entre les modèles ne doit pas être pris à la légère et indiquent, selon nous, que quelque chose de très fondamental concernant la nature de l'électron est mis en avant par cette étude.

Key words: Zitterbewegung; Electron Models; Generalized Holographic Model; Vacuum Fluctuations; Quantum Gravity.

I. INTRODUCTION

Our understanding of matter is linked to the evolution of our understanding of energy and the relationship between both; its paramount owed to Albert Einstein and his famous $E = mc^2$ equation. When matter was only understood in terms of its first known physical states, energy was obtained by combustion of flammable material such as wood, carbon, gas, and later oil. Such was the level of our understanding of nature at the time, giving the first hints of energy extraction from mass. The industrial revolution started during the period of vapor machines through heat, developing further with the transition to electricity; a "charged fluid" capable of performing work, and thus, becoming a source of utilizable energy. Thanks to electricity and electromagnetism, wired and unwired transmissions appeared, going from the invention of the radio, telephone, and television, up to the internet. Our lives have changed dramatically since then.

Comprehension of matter went further, or one might say, deeper, when matters' atomic structure was elucidated. The atom and its different models took the stage, and nuclear energy appeared together with its humongous power. Counterintuitively, the smaller we went in volume, larger the amount of energy that could be extracted, suggesting that a sort of internal pressure by confinement increase, increases the energy available when reaching such density gradients.

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The advantage of extracting much more energy per matter volume has given rise to unstoppable progress at all levels of human life. But revolution not only came as a technological application; the black body radiation problem and the photoelectric effect gave birth to quantum theory, the science of the atomic and subatomic scales. Quantum theory came to defy the laws of the macroscopic world, and since then no leading approach has been able to conciliate the laws of quantum theory with relativity and gravity into a unified theory; a fundamental piece of the puzzle is still missing in the so called "unification problem" or theory of everything.

It was found that energy in matter was structured at the atomic level and that energy exchange with the electromagnetic field occurred through discrete packets called photons; atomic structure and light-matter interaction were quantized. The origin of such quantization remains unknown, and it is directly related to one of the most important values in physics known as the Planck constant h. With the emergence of quantum mechanics and the leading Copenhagen interpretation, there is no doubt that it is the very nature of our reality what is being inquired. The philosophical and practical implications of the probabilistic approach embedded in the Copenhagen interpretation have led us to very uncomfortable scenarios that go beyond the scope of this review.

Technology has reached unbelievable achievements intertwining theory and applications in areas such as nanotechnology. Interestingly, nanotechnology could be defined as the multiple possibilities of surface-to-volume ratios or relationships between geometries in nanoparticles. The nanoscopic regime has determined the elements physical-chemical properties (such as conductivity) in terms of their nanoparticle size and shapes, equivalent to increasing the periodic table in an additional dimension; properties fixed at the macroscopic scale become shape and size dependent at the nanoscale. Yet, the building blocks of these elements, mainly electrons and protons, remain a puzzle. Besides the intriguing dependence of a nanomaterials' physical-chemical properties with respect to its surface-to-volume relationships,[1,2] is the observation that more than 99.999 999…% of volume in mass is considered empty space … is it the case? What if vacuum was another kind of material, as suggested by Wilczek in his "materiality of the vacuum"?[3]

The discovery, implications, and applications of such fundamental particle as the electron, which discovery predated the discovery of the atom itself, have been a beautiful endeavor for humankind. One realizes that we could hardly become fully aware of its nature, and yet, there is so much that has been deciphered about it. The aim of this work is to condense in a short review, a collection of perspectives and semiclassical descriptions of one of the most important personages of the material realm; the electron, which is 1836 times less massive than the other most important personage of the atomic constituent's arena; the proton.

In the literature, we find semiclassical descriptions of the electron's nature and structure, including the fully relativistic, i.e., non-classical approach by Wilson, which are an alternative to the leading Quantum Electrodynamics and the Copenhagen interpretation, and that can account for many features of the electron, even if indispensable predictions such as the anomalous magnetic moment are work in progress in some of these backstage theories. The models presented hereafter go beyond the description of the electron as a structureless nondimensional point like charge with momentum and position determined by a probabilistic interpretation of the wavefunction, described in terms of an electronic density cloud.

These models share features in common that provide useful insights concerning the nature of the electron; for instance, they all consider *zitterbewegung* (a light-speed trembling-along-the-way electron motion, abbreviated here as zitter) to be a real oscillatory motion of the electron.

On the one hand, we have the zitter fluctuations happening at light speed c and associated only to particles as predicted by Dirac's equation, and on the other hand we have the zero point energy—zpe—associated with the vacuum fluctuations addressed by QFT/QED and experimentally proven by the Casimir effect, so clearly, a very important link is missing in the current state of particle physics and QFT, since one could reasonably suppose there should be an unambiguous connection between both type of fluctuations, regardless of the discussion on the nature of zitter and the zpe; if real oscillations in (of?) the quantum vacuum, or not. Additionally, if zitter was a real physical rotation, then it would imply that the electron has inner structure, while QFT considers the electron a fundamental particle because it has no inner structure; it is a point-like particle as established by the Standard model.

Given that zitter and zpe are not yet unified in a consented framework showing their relationship, our review addresses in Section II the zitter models proposed by physicists who belief that zitter is a real rotation which would originate the spin and magnetic moment of a particle, and subsequently, in Sections III–VII, the models that address the zero-point field as real rotations of the quantum vacuum and that would give rise to particles' structure and properties. We apologize if such separation is confusing to the reader, but we must remark that the topic is not clear in current mainstream approaches.

The zitter models included in Section II are: Section II A *Schrödinger's interpretation of zitterbewegung*, Section II B *Zitterbewegung interpretation of quantum mechanics,* from David Hestenes, Section II C *The electron as a 2D harmonic oscillator*, from Jean L. Van Belle, Section II D *The ring and the helical electron model*, from Oliver Consa, Section II E *The Superluminal quantum model of the electron and positron* from Richard Gauthier, and finally, Section II F *The zitterbewegung electron model and Occam's razor*, by Giorgio Vasallo *et al.*

The origin of zitter should probably be traced back to the vacuum fluctuations, thought to be the source of the electron inner structure, charge and mass as proposed in models of Section III. *Stochastic electrodynamics*, from Haisch *et al.*, Section IV. *Dirac electron model in the Planck Vacuum Theory*, from William Daywitt, Section V. *Dirac Electron Model and QED-P* from James Wilson, Section VI. *Dirac Kerr Newman electron model*, from Alexander Burinskii and Section VII. *The holographic mass electron model*, from Amira Val Baker *et al.*

Following the line of reasoning that connects them, we conclude with Val Baker's *et al*. generalized holographic model (GHM) applied to the electron. The GHM has given the most striking results by predicting the proton muonic radius (most recently confirmed by the latest electronic hydrogen measurements from Ref. 4) within 1σ standard deviation and with no adjusting parameters. In the discussion section, we compare and show why these models for the electron can be directly and unambiguously related.

It is our belief that such coincidence should not be taken as a random event and that something very fundamental concerning the nature of the electron is being unfolded. These theories seem to convey a unified view, while the leading approaches are struggling to reach unification. Considering the huge amount of infrastructure, financial resources, and brilliant researchers engaged so far, it seems urgent the incorporation of new theories, and the reconsideration of not so new theories, to complete the puzzle at this level.

The theories explored in this review have reached stages and results which are very promising.

A. Considerations about the Electron and the Planck Units

While electricity and electromagnetism had been widely explored by many visionaries since 1600 (including William Gilbert, Otto von Guericke, Robert Boyle, Alessandro Volta, Hans Christian Ørsted, André-Marie Ampère, Michael Faraday, Georg Ohm, and James Clerk Maxwell, just to name the most known), it was the Irish physicist George Stoney who introduced the concept of a fundamental unit of electricity that later in 1874 he coined Electrine, an atom of electricity. In 1881 Stoney proposed the term electron to name this unit of charge.[5] At the time, the particle we now call the electron was not yet discovered and the difference between the particle *electron* and the unit of charge *electron* was still blurred. When the name *electron* was assigned to the particle, the unit of charge e lost its name. However, the unit of energy electron-volt reminds us that the elementary charge was once called *electron*.[6]

Stoney made significant contributions not only on the conception and calculation of this unit, but also in cosmological and gas theory physics. His work laid the foundations for the discovery of an electron outside the material, performed by J. J. Thomson in 1879 at the Cavendish Laboratory in Cambridge University.

1. The Stoney scale and the Planck scale

It is worthwhile noticing that the Planck scale, considered the most suitable scale for a unified theory, was anticipated by Stoney who stablished the first example of natural units of measurement designed so that certain fundamental physical constants served as fundamental units. The *Stoney scale* and the *Planck scale* refer to size and energy scales where electromagnetic, quantum processes and gravity occur simultaneously and so calling for a unified theory of physics. Weyl attempted to construct such a theory from the Stoney scale, associating a gravitational unit of charge with the Stoney length, what apparently inspired Dirac's fascination with the large number hypothesis initially proposed by Weyl[7] and lead him and others to believe that the size scale and the forces scale are intrinsically connected.

TABLE I. Stoney units.

Quantity and (dimension)	Expression	Value (SI units)
Length (L)	$l_S = \sqrt{\frac{G}{c^4}\frac{e^2}{(4\pi\epsilon_0)}}$	$1.380\ 68 \times 10^{-36}$ m
Mass (M)	$m_S = \sqrt{\frac{e^2}{G\ (4\pi\epsilon_0)}}$	$1.859\ 21 \times 10^{-9}$ kg
Time (T)	$t_S = \sqrt{\frac{G}{c^6}\frac{e^2}{(4\pi\epsilon_0)}}$	$4.605\ 44 \times 10^{-45}$ s
Electric charge (Q)	$q_s = e$	$1.602\ 18 \times 10^{-19}$ C
Temperature (T)	$T_S = \sqrt{\frac{c^4 e^2}{G\ (4\pi\epsilon_0) k_B^2}}$	$1.210\ 28 \times 10^{31}$ K

Thirty years later, Planck proposed a new set of units by renormalizing the reduced Planck constant ħ instead of the elementary charge e that Stoney had normalized to one. Evidently, if the Stoney length or Planck length were defined as the minimal length, they cannot be both minimum cutoff lengths simultaneously because they do not have the same value. For this reason and given the fact that the Planck constant is associated with energy, and more precisely with the vacuum energy, below Planck length it has been assumed that space and time are not defined; the Stoney length—being one order of magnitude smaller than Planck length, have fallen out of use since then.

In Stoney's units, the constants normalized to one are $c = G = k_e = e = 1$, where c is the speed of light, G is the gravitational constant, e is the electric charge, and k_e is the Coulomb constant. In Planck units the constants normalized to one are $c = G = k_e = \hbar = K_B = 1$, where K_B is Boltzmann constant. For sake of completeness, we introduce both Stoney and Planck units in Tables I and II.

The SI units were redefined in May 2019[8] during the 144th anniversary of the Metre Convention. The most important of the redefined units was the kilogram, which is now defined in terms of the Planck constant h that has been measured with extraordinary precision in recent years. Its agreed

TABLE II. Planck units.

Quantity and (dimension)	Expression	Value (SI units)
Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\ 255(18) \times 10^{-35}$ m
Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176\ 435(24) \times 10^{-8}$ kg
Time (T)	$t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\ 245(60) \times 10^{-44}$ s
Electric charge (Q)	$q_P = \sqrt{4\pi\epsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}}$	$1.875\ 545\ 956(41) \times 10^{-18}$ C
Temperature (T)	$T_P = \frac{m_p c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G\ k_B^2}}$	$1.416\ 785(16) \times 10^{32}$ K

value has been set as $6.626\ 070\ 15 \times 10^{-34}$ kg m^2 s^{-1}, allowing researchers to make precise mass measurement using equipment such as the Kibble balance.[9] The MKS system is now completely described in terms of vacuum and quantum regime properties, which are fundamental agents.

Having all units converged to the Planck constant, the only remaining issue is the limitation posed by the gravitational constant G by which all Planck units presented in Table II depend upon. G is the constant with lower accuracy of 10^{-5}, becoming the limiting factor. Now that the Planck constant has been fixed to a more accurate value and that the units of mass depend on it, the increase in G accuracy will only depend on achieving the solution to quantum gravity.

Between 1879 and our present time, many attempts to decipher the electron inner structure have emerged, inspired by de Broglie's idea of applying Einstein-Planck relationship $E = hf$ to a particle. This empirically derived relation that Max Planck obtained in 1900 to solve the black body radiation problem, was five years later determined to be the quanta of the electromagnetic field (photon) by Einstein, providing a particle nature to the radiation field. De Broglie's audacious idea granted a wavelike nature to the massive entity through a "natural frequency" given by $f = mc^2/\mathrm{h}$ coming from equating Einstein's $E = mc^2$ and Planck's $E = \mathrm{h}f$. When applied to the electron, this natural frequency $f_e = m_e c^2/\mathrm{h}$ matches the value of the electron Compton frequency, and this was interpreted at the time as a real oscillatory motion.

As summarized by Consa,[10] an electron vibrating at this constant frequency f_e generates an electric current equal to the electric charge at that frequency ($I = ef_e$) that induces a magnetic field according to Maxwell's equations. The prediction of a magnetic moment associated with this oscillation was measured with great precision, being approximately the Bohr magneton value $\mu_e = \mu_B = e\ \hbar/2m_e$ where m_e is the electron mass. The Bohr magneton is the magnetic moment corresponding to a unitary charge that rotates with angular momentum equal to the reduced Planck constant $L = m_e r v = \hbar$. The relationship between the magnetic moment and angular momentum is called the gyromagnetic ratio g and has the value $g = e/2\, m_e$ consistent with the magnetic moment generated by an electric current rotating on a circular surface of radius r.

By applying external magnetic fields, the gyromagnetic ratio of the electron can be observed experimentally. Such is the case in the Zeeman effect or in the Stern–Gerlach experiment, where we find that $E = gB$, being B the magnetic field. This suggests that the electron is a rotating sphere with a frequency equal to the Compton frequency generating an angular momentum or spin equal to the reduced Planck constant, and a magnetic moment equal to the Bohr magneton. Being this the case, the speed rotation of the electron is higher than the speed of light. Additionally, the magnetic moment of the electron is experimentally slightly higher than the Bohr magneton (the so-called anomalous magnetic moment predicted accurately by the standard model by inserting a contribution coming from the vacuum fluctuations). This last could suggest that there is an inner structure to the electron, as the authors in this review will address from their own perspectives.

To avoid superluminal speeds, quantum electrodynamics defines the electron as a static point charged particle without rotation, where the magnetic and angular momentum of the electron would have intrinsic values not associated with any real movement. This raises a deep concern, as remarked by Vasallo,[11] because point-like shaped elementary particles with intrinsic properties as mass, charge, angular momentum, magnetic moment, and spin are not possible according to the laws of mechanics and electromagnetism. Other incongruences are that a magnetic moment must necessarily be generated by a current loop that cannot exist in a point-like particle, and the electric field generated by a point-like charged particle should have an infinite energy.

As Vasallo claims,[11] it is imperative to find an alternative realistic approach that fully addresses these very basic problems, and the *Zitterbewegung* interpretation of quantum mechanics, according to which charged elementary particles can be modeled by a current ring generated by a massless charge distribution rotating at light speed along a circumference whose length is equal to the particle Compton wavelength, seems to be a good assumption.

The very first attempts to have a geometrical description of the electron was that of a rotating sphere, but this gives superluminal speeds and for this reason the point particle view has prevailed, while other models have not been pondered. Nevertheless, as Consa posits, there are alternative models that can prevent superluminal speeds. For instance, the ring or the helical models have been proposed[10] and are giving results in agreement with experimental predictions.

In the ring electron model, an electric charge circulates at the speed of light in a ring-shape form of radius equal to the Compton wavelength of the electron λ_C. The circular motion of this electric charge causes the magnetic moment of the electron. Furthermore, in 1952 Kerson Huang proposed a semiclassical interpretation of the Dirac equation,[12] whereby zitter is the mechanism that produces the spin of the electron, and this spin produces the magnetic moment of the electron. Renowned researchers as Asim Barut and David Hestenes have worked on this "Zitterbewegung electron model," and both models offer a semiclassical alternative to the current electron model of Quantum Mechanics. Although zitter is thought to be the cause for the electron spin, this oscillatory motion is often considered in relativistic scenarios and mostly neglected elsewhere.

In this regard, the fully relativistic work by Wilson[15–19] offers an interesting solution based on his Dirac Electron Model (DEM) in which the electrons' center of charge (CoC) position spins around its center of mass (CoM) as a set of three one dimensional harmonic oscillators with zitter period of 1.5×10^{-21} s. His model will be addressed in Section V.

B. Our current electron mass equation

The electron is considered a fundamental particle in the sense that it has not shown to have an inner structure. It is notorious that in quantum electrodynamics (QED), the field of quantum mechanics describing electrons and their interactions with photons, there is not a definitive description of either electron or photon. We have a very good idea of their effects and interactions between them, but very little is known about their inner nature.

One of the most precise values we have for the electron is its mass, determined using penning traps.[13] These measurements are extremely precise, with a relative uncertainty of order of 10^{-8}. The standard definition for the electron mass, in agreement with the updated recommended CODATA 2018 value is given by the following expression:

$$m_e = \frac{2R_\infty \mathrm{h}}{c\alpha^2} = 9.10938370(15) \times 10^{-28} g, \tag{1}$$

where R_∞ is the Rydberg constant, h is Planck's constant, c is the light speed, and α is the fine structure constant.

Equation (1) combines first principles calculations with empirical models for Rydbergś constant, resulting in an accurate value for the electron mass. However, Eq. (1) is not entirely derived from first principle notions, and thus it is far from providing an insight to what the electron is.

Although the mass m_e of the electron has been well stablished, the radius is another story, and the size of the electron has been long debated. From a purely electromagnetics derivation, classical physics provides that the electron radius r_e (in CGS units) is[14]

$$r_e = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13} \text{ cm}.$$

While the electron Compton radius, which is the one obtained from scattering experiments and so is taken as the photon scattering radius of the electron, is given by (in CGS units)

$$r_C = \frac{e^2}{\alpha m_e c^2} = 3.86 \times 10^{-11} \text{ cm}$$

being larger than r_e by $1/\alpha \cong 1/137$, where α is the fine-structure constant. Compton radius is a boundary condition, i.e., it determines a spherical volume; within this volume (radius smaller than r_c) the possible appearance of electron-positron pairs make it inappropriate to consider it classically. The onset radius for electron-position pair production is a crucial parameter in the Dirac theory of the electron.

The origin of the fine structure constant α is still a mystery, we could say, it is the most fundamental constant as it does not depend on the units chosen, and on the contrary, it seems like all physical properties would depend on it. A second mystery is the origin of the *zitterbewegung* or "trembling along the way" electron motion: the lightspeed oscillatory motion of the electron which is superposed with its translational motion.

The advances to our understanding of nature would be mind-expanding if all physical properties (mass, radius, α, *Rydberg constant…*) emerged from a different description starting from first principles notions. In Sections II–VIII, we will address the models that head in such direction arriving to a model that provides a coherent explanation that could be definitive at this level.

II. THE *ZITTERBEWEGUNG* OR TREMBLING ALONG THE WAY ELECTRON MODELS

The German word *zitterbewegung*—zitter—means trembling along the way, and it was coined for the first time by Erwin Schrödinger in 1930 when studying the solutions of Dirac equations for free relativistic electrons. When analyzing the behavior of the wave packets, Schrödinger found an oscillatory term with an amplitude of the magnitude of the Compton wavelength and frequency of $2\,m_e c^2/\hbar$. This zitter character of the electron found by Schrodinger suggested that the electron performed an extremely high-frequency circular motion, and furthermore, that the radius of this theoretical circular motion and the electron photon scattering radius were the same. This makes sense since the Compton scattering radius is taken as the effective radius of the electron charge.

Due to the extremely high speed of this oscillation, mainly the speed of light, the zitter motion of a free electron would be impossible to detect experimentally and various experimental simulations of the effect have given insight. Recently, it has been shown theoretically that electrons in III-V semiconductors are governed by similar equations in the presence of spin-orbit coupling,[21–23] where a small energy splitting up to 1 meV result in zitter at much smaller frequencies, being experimentally accessible as an AC current that would demonstrate the zitter of electrons in a solid.[24] Zitter has also been used to explain the nontrivial behavior of the conductivity at zero temperature in graphene[25] where the importance of considering this electron effect has been pointed out. Among other consequences, the zitter behavior of the electron is used to produce the Darwin term for the hydrogen atom, which plays the role in the fine structure as a small correction of the energy level of the s-orbitals.[26] Also, the electron channeling and internal clock experiments are explained in terms of zitter.[27,28]

Many interesting features emerge from the zitter behavior that are worth-like noticing. The electron would have to be massless at the level where these fluctuations occur, since zitter happens at the speed of light. Therefore, mass would have to appear at some outer, external frame or level of motion. The fluctuations could also explain a smearing out of the average position over a Compton radius volume, which would give a physical interpretation to the wave function and the associated probability density. And this is somehow supported by scattering experiments, which indicate that the electron is far smaller than its Compton size, being more of a point-like charge.[29] As Wilczek pointed out "An electron's structure is revealed only when one supplies enough energy […] at least 1 MeV, which corresponds to the unearthly temperature of 1010 kelvin" below which it "appears" point-like and structure-less.[30]

Simulations have shown that if a massless fluctuating point particle is accelerated in an electric field, zitter acquires a helical motion suggestive of spin.[12] Therefore, it has been claimed by some researchers that zitter is the origin of spin, fact that could be supported in the relativistic regime through Schrodinger's interpretation of Dirac's solution for the velocity operator of the free electron. However, others

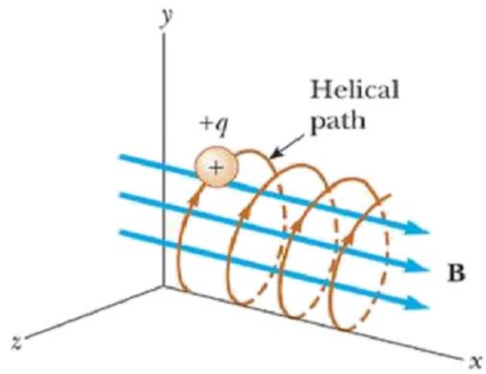

FIG. 1. (Color online) Electron motion in a homogeneous external magnetic field B follows a helical path. Image from Ref. 10, published under CC-BY license by IOP Publishing Ltd.

such as Barut and Zanghi,[31,32] Hestenes,[33] Huang,[12] Gauthier,[34] Consa,[10] Vassallo,[35] Knuth,[36] and Puthoff *et al.*[37] suggest that there must be another alternative classical explanation for the zitter motion, since spin is present always and not only in the relativistic domain.

Zitter models differ from each other though they all come to the same conclusion, which Consa summarized in his Principle of Helical Motion:[10] "A free electron always moves at the speed of light following a helical motion, with a constant radius, and with the direction of movement perpendicular to the rotation plane." This helical motion of the electron is analogous to the observed motion of an electron in a homogeneous external magnetic field, as shown below (Fig. 1 taken from Ref. 10).

Zitter suggests profound links between the zero-point energy, the mass-energy relationship of matter and the quantum properties of particles, all of which will be explored in Sections II A–II F.

A. Schrödinger's interpretation of Dirac's free electron model

Relativistic quantum mechanics predicts that free Dirac electrons exhibit a rapid trembling motion, even in the absence of external forces. The leading explanation for this motion is the interference between the positive and the negative-energy solutions of the Dirac equation separated by 1 MeV which create oscillations, fluctuations, or circulatory motion of the electron at extremely high frequencies—speed of light—which are out of reach experimentally. This confines the effect within the relativistic frame but then as Hestenes points out "how could the *zitterbewegung* be the origin of spin, which is surely significant in the nonrelativistic domain?".[38]

As Knuth remarks,[36] the origin of zitter traces back to Schrödinger and Breit. Both found an oscillatory term when analyzing the free-particle wave packet solutions of the Dirac equation for fermions, where the eigenvalues of the velocity of a particle described by wave packet solutions to the Dirac equation were strictly $\pm c$ the speed of light, implying that all fermions zig-zag back-and-forth at *only* the speed of light. Since massive particles must move at speeds less than the speed of light, zitter challenges relativity, hence, this result for the electron with mass m_e was reinterpreted through the existence of an interference between positive and negative electronic energy states oscillating with circular frequency[38]

$$\omega_e = \frac{2m_e c^2}{\hbar} = 1.55 \times 10^{21}\,\mathrm{s}^{-1}.$$

This angular frequency at which a particle "zitters" is then given by twice the de Broglie electron clock rate as determined in his 1924 dissertation.[36]

Schrödinger interpreted this as a fluctuation in the position of the electron with amplitude $\lambdabar_e$,

$$\lambdabar_e = \frac{c}{\omega_e} = \frac{\hbar}{2m_e c} = 1.9 \times 10^{-13}\,\mathrm{m},$$

which corresponds to a characteristic length which is half the reduced Compton wavelength.

For an electron moving at c about a mean position, the angular momentum is

$$\lambdabar_e m_e c = \frac{\hbar}{2},$$

which is naturally interpreted as the spin angular momentum of the electron. The oscillation between positive and negative energy states would explain the origin of the electron spin, and it is also related to electron-positron pair creation and annihilation,[38] reason why zitter would be a purely relativistic phenomenon. This would also imply Dirac theory to require the finite speed of a particle ($v < c$, including at rest $v = 0$), to be described as the average speed of a particle that is zig-zagging back-and-forth at the speed of light. Therefore, any observed macroscopic speed is a resultant drift velocity.[36]

As it stands, Dirac's fully relativistic equation in the form originally proposed by Dirac[39] for a free electron

$$\left(\beta m_e c^2 + c\sum_{k=1}^{3}\alpha_k p_k\right)\psi(\boldsymbol{r},t) = i\hbar\frac{\partial\psi(\boldsymbol{r},t)}{\partial t}$$

describes all spin-1/2 massive particles, and it is consistent with the principles of quantum mechanics and the theory of special relativity. The relevance of this equation relies upon three critical aspects: it accounts for the fine structure of the hydrogen spectrum, it predicts the existence of antimatter, and it provides a theoretical justification for Pauli's phenomenological theory of spin. The only controversial feature of this equation is that it depicts subatomic particles oscillating at the speed of light c; a movement that superposes to its translational motion.

Because this physical oscillation of mass would violate special relativity principles, this fluctuation in Dirac's theory was interpreted instead as a fluctuation between positive and negative energies in the Dirac sea. The Dirac sea establishes that all negative energy states are occupied and Pauli's exclusion principle forces any additional electron to occupy the positive states, which are also supposed to be all occupied. Dynamically, the interaction of both energy states happens via the quanta of the electromagnetic fields, photons that are continuously absorbed and emitted by electrons leaving holes behind or falling into the Dirac sea and seemingly annihilating. Virtual particles that pop in and out of existence as a particle-antiparticle pairs creation/annihilation process, are supposed to be the origin of vacuum fluctuations as well.

In such a scenario there should be a direct link between zitter and the zero-point energy fluctuations of the quantum vacuum—zpe—which are always present. In modern terms, the Dirac sea can be understood via quantum field theory—QFT—as a sum of creation and annihilation operators for the Dirac spinor. And yet, zitter is traditionally associated only to the Dirac equation at the level of quantum mechanics, and very little has been done in QFT to address it,[40] while QED introduces the concept of vacuum fluctuations following the quantization of the electromagnetic field, i.e., an EM field composed of quantum harmonic oscillators. The quantum vacuum state is the fundamental state of the electromagnetic field, so basically, the complex interaction of the Dirac sea with the quanta of the EM fields produces a diversity of phenomena, such as the spontaneous emission of an electron interacting with this fundamental field.

The Dirac sea predicted the existence of antimatter; an electron in the Dirac sea absorbing energy would leave a hole with the same mass as the electron but with a positive charge, a positron. Antimatter was later confirmed by Carl Anderson[41] in 1930, validating such a view. Therefore, the depiction of zitter as a real mechanical oscillation has no place in QFT/QED and it is only considered as the fluctuation of the Dirac sea in relativistic scenarios, while mostly neglected elsewhere. Meanwhile, the zpe are always present, not just in relativistic domains. This last raises concerns among the authors who consider zitter a real oscillation, because in their view zitter would be the origin of spin and magnetic moment of particles, which are present all the time and not just in relativistic domains.

Since zitter has never been directly observed, it is barely mentioned in physics texts. However, the experimental observations mentioned earlier,[24–28] together with the ones listed by Knuth, such as the electron channeling experiments in silicon (that detected a resonance at the electron clock rate[42–44] and which was explained theoretically by Hestenes[42]) the observed zitter-like behavior in trapped ions[46] and atoms in a Bose-Einstein condensate,[47,48] among many others, are evidence that it may be a real effect that could revolutionize how we think about mass and motion, because if particles zitter, the concept of rest would be reasonable only in the sense of long-term averages. This later affects the concept of a rest frame, which is fundamental to spacetime physics. This would also relate to Haug's work proposing that mass is a collision rate.[49] As Knuth points out, zitter would describe the kinematics of the particle but not the dynamics, since it is not well-understood what would cause the particle to undergo reversals in direction.

The interesting work of Knuth[36] considering what happens if zitter is described properly, shows that any finite speed less than c, including the state of rest, only makes sense as a long-term average that can be thought of as a drift velocity. He considers this idea that the observed velocities of particles are time-averages of motion at the speed of light, and he demonstrates how the relativistic velocity addition rule in one spatial dimension is readily derived by considering the probabilities that a particle is observed to move either to the left or to the right at the speed of light.

Although zitter was not fully understood or described, the idea was left aside, and it was not until the 80's that Hestenes and others revisited it. They have developed alternative interpretations and scenarios to release this relativistic regime constraint imposed by Schrödinger's interpretation on the *zitter* behavior.

B. Zitterbewegung interpretation of quantum mechanics

As David Hestenes points out, there is no definitive and satisfactory physical explanation to the zitter behavior shown by the Dirac equation, and the implications of the zitter motion are still debated. In this sense we can state that Dirac theory remains incomplete, and it will not be completed while this feature remains unsolved. When attributing an interpretation to the oscillation, Hestenes proposes mainly three scenarios;[38] (1) zitter is an unreal theoretical outcome from an incomplete theory, or (2) random electron-positron pair creation and annihilation produce a zitter disordered motion of the electron (this is the mainstream approach), or (3) zitter it is a real circular motion of the electron, which at rest is associated with the plane waves description, and it is helical when the electron is moving translationally (this last is associated with the wave packet). The third view implies that the electron spin can be identified with the orbital angular momentum. All authors collected in this review support this option because it proposes a physical mechanistical explanation for the electron spin, that seems reasonable and that has provided interesting results.

One of the most developed models for zitter behavior is that from Hestenes,[33,38,42,45,46,50,51] at Arizona State University. He proposes a simple model of the zitter which is consistent with the Dirac theory. Very roughly speaking, instead of a fluctuation between energy states of the Dirac sea, his idea is to consider zitter motion a real physical, circular motion of a point particle, for which he interprets the mathematics differently and reconsiders the function describing a free electron in Dirac theory. Mainly, he states that the complex phase factor in the wavefunction describing the electron would represent, literally, the spin of the electron. Until now, the sign in the phase factor is considered a matter of convention since both are solutions to the same differential equation when taking the norm of the wavefunction, which is the quantity related to the observable and measure. Nevertheless, if this phase factor sign is more than a convention and has a physical consequence, this would have far-reaching implications because the complex phase factors play a critical role in quantum mechanics. The most important implication being that spin would be explained mechanistically, which would imply a more classical, deterministic description of an object that is usually considered a probability distribution function.

As Hestenes states:[38]

> "it implies that Schrödinger's original wave packet oscillation is merely an epiphenomenon revealing the zbw (zitter) periodicity which was already inherent in the complex phase factors of both electron and positron plane wave states. The essential feature of the zbw idea is the association of the spin with a local circulatory motion characterized by the phase factor. Since the complex phase factor is the main feature which the Dirac wave function shares with its nonrelativistic limit, it follows that the Schrödinger equation for an electron inherits a zbw interpretation from the Dirac theory. It follows that such familiar consequences of the Schrödinger theory as barrier penetration can be interpreted as manifestations of the zbw."

In Hestenes' model, the electron is a massless point particle executing circular motion in the rest system. He refers to the radius of the circular orbit as the zitter amplitude, while the circular frequency of the motion is called the zitter frequency. The center of orbit is the zitter center and the angular position of the electron on the circular orbit is called the zitter phase angle.

Using the space–time algebra,[50] Hestenes defines the "canonical form" of the real wave function ψ,

$$\psi(x) = \left(pe^{i\beta}\right)^{1/2} R,$$

where $\psi(x)$ is a space–time scalar function representing a probability density, proportional to the charge density, i is the spatial bivector ($i = \gamma_2\gamma_1$), for a plane in space in both the energy momentum operators $\boldsymbol{p}$ and in the complex phase factor of the wave function. This would be the spin plane in which the zitter circulation takes place. $\beta = \beta\ (x)$ function represents the value of a rotation phase in the plane $\gamma_2\gamma_1$ and R is a rotor valued function that encodes a Lorentz transformation. Therefore, the phase factor $\beta\ (x)$ represents a physical rotation, a zitter rotation and the $\boldsymbol{p}$ operator acting on this phase factor computes the rotation rates of the phase in time and space directions, identifying them with the energy and momentum, respectively.

This means that the imaginary unit i representing a generic undefined complex plane in Dirac's canonical wave function, is replaced by a bivector that generates rotation in a well defined space-like plane. This extended feature reveals a geometric meaning for the imaginary numbers in the wave functions of quantum mechanics.[51] In this frame, the probability density function is associated with a point-like shaped charge, and by applying the relativistic time dilation to the zitter period, Hestenes predicts a zitter angular frequency that slows down as the electron speed increases.

Through this reformulation of Dirac theory, Hestenes considerations of the wavefunction gives a physical interpretation for the complex phase factor Dirac's wave function, providing further coherency to the physical interpretation of Dirac's theory. Just as Schrödinger, Hestenes' interpretation arises from analyzing the structure and motion of the electron in the solutions of the Dirac equation, but he employs a reformulation of the theory in terms of the real *spacetime algebra* which he has developed and discussed in depth.[50–58] His reformulation reveals a geometric structure in Dirac theory which remains hidden in the conventional definition and interpretation; mainly that the Dirac wave function describes the kinematics of electron motion and how this depends on the definitions of the observables in the theory.

In a study of plane wave solutions of the Dirac equation he gets to the essential new idea: a simple change in the definition or interpretation of the electron velocity implies that "all components of the electron wave function (including its phase) directly describe kinematical features of electron motion."[45]

In Hestenes' zitter interpretation of the Dirac Theory, the magnetic origin of the zitter is due to the self-interaction of the electron with its own electromagnetic field, and he shows that this interaction has the form of a Larmor precession energy. Therefore, and like Haisch et al., the rest mass of the electron would be kinetic energy coming from the magnetic self-interaction which gives the electron its inertial properties.[55] The "flywheel-like nature of this inertia may be the ultimate origin of spin dependence in electron scattering."[45]

Hestenes calls his approach "the zitterbewegung interpretation of quantum mechanics" because the most common features of quantum mechanics can emerge from the zitter behavior. For example, Heisenberg uncertainty principle can be attributed to the zitter effect because an electron cannot be confined to a region smaller than a Compton wavelength.[54,57] Hestenes shows that zitter fluctuations in momentum p would produce the quantum barrier penetration, while a shift in the zitter phase would produce the Aharonhov-Bohm effect.

The internal oscillation of the electron could be source of an electromagnetic field fluctuating at zitter frequency, which would be too high to be detected directly, though it could provide a mechanism for explaining the Pauli principle.[54] Detection of some zitter effect which is not already explained by the Dirac theory or quantum electrodynamics, has been proposed by Hestenes.[58]

Given the relevance of all these features which are still work in progress, the zitter interpretation must be explored and developed thoroughly. For this, or any other zitter model to be fully accepted and incorporated as the real mechanism, it would also have to predict behaviors which are already accounted by QED and related theories, such as the anomalous magnetic moment.

The authors explored in this review propose interesting models to explain *zitterbewegung* in semiclassical ways, and some have addressed the anomalous magnetic moment, including Consa,[10] Vassallo,[11] Wilson,[15] and Van Belle.[59]

We start with Van Belle's model, hereafter.

C. The electron as a two-dimensional harmonic oscillator

In a similar view to Hestenes', Van Belle[59] proposes a very intuitive and straightforward interpretation for the zitter motion which could also provide a more realistic interpretation for QED. To properly address the model, we start with the very basics, linear and angular velocity. Relevant features of this very well-known system are key to the various theories presented, so a short recall of circular motion will refresh the notions concerned and it will be useful to have them handily accessible. This section also allows introducing the general physical concept behind a realistic interpretation of zitter that different authors besides Van Belle support.

A circle of radius r has a circumference $C = 2\pi\, r$; this is the arc length for a 2π rotation from a center located at a distance r, the angle of rotation being $\phi = 2\pi$. Following this rationale, for an arbitrary angle of rotation ϕ of a reference particle (black dot in Fig. 2), the distance or corresponding arc length l during the rotation is given by $l = \phi\, r$, as shown in Fig. 2.

The arc length l has an associated linear velocity v and time required to travel such a distance. This velocity for linear motion is given $v = l/t$. The angular velocity or speed at which the ϕ angle changes is known as angular frequency ω. In constant motion, $\omega = \phi/t$, in radians/time. Equations $l = \phi\, r$ and $\omega = \phi/t$ are related by the angle ϕ through the following expression $\phi = \omega\, t = l/r$ such that we can rearrange terms to obtain $\omega = v/r$ or $v = \omega\, r$.

When the particle has completed a $\phi = 2\pi$ rotation angle, the total arc length for the circumference of radius r is $C = 2\pi r$ and the dot has completed a cycle. The time taken to complete a cycle, or period T, is such that $\omega = 2\pi/T$ and this is also written as $\omega = 2\pi f$ where f is known as frequency and is the inverse of the period $f = 1/T$. If the period T is the time it takes to complete a circumference or cycle, then frequency f is the number of cycles or circumferences per unit of time (second).

In Van Belle's depiction of zitter, the point particle is an electron following the circular motion of the pointlike charge which is driven by a tangential force. The free electron depicted as a pointlike charge in an electromagnetic orbital oscillation of radius r, rotates at tangential light speed velocity of c ($v = c$). At the same time if $v = \omega\, r$ then $c = \omega\, r$. We can insert this speed in Einstein's equation $E = mc^2$ giving $E = m\,(\omega\, r)^2$. This last is relating not only energy with a rest mass m but it is also calling for an internal mass-energy or vibration contained in the frequency ω which could have some physical meaning. The speed of light c in $E = mc^2$ would then be a linear or tangential velocity v directly related to zitter motion. When the electron has completed a $2\pi\, r$ rotation, and since its tangential speed is $c = \omega\, r$, then $r = c/\omega$ and if the zitter frequency is given by Planck's energy-frequency relation, then by substituting $\omega = E/\hbar$ and $E = mc^2$ we have that $r = \hbar/mc$ which can be defined as a Compton radius $r_C \approx 0.386 \times 10^{-12}$m, and Compton wavelength $\lambda_C = 2\pi\, r_C$. This means that $r = r_C$ and the circumference or complete rotation is $\lambda_C = 2\pi\, r_C$ ($\lambda_C = C$ in Fig. 2). What Van Belle defines here as r_C is directly related to the radius of curvature of the electron history proposed by Hestenes.[36] As Van Belle remarks, the bold assumption $c^2 = r^2\omega^2$ interprets spacetime as a relativistic aether because the most direct implication of Einstein's $E = mc^2$ is that the ratio between the energy and the mass of any particle is always c^2. The internal frequency ω^2 can be related to harmonic oscillators through expressions $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$, though these last introduce at least two degrees of freedom, while c^2 is equal to E/m for any particle. This means that we can describe the aether in terms of the standard features of electric circuts: resistance, inductance and capacitance, as well as and the stiffness of springs and the masses present, keeping in mind that this applies only to our spacetime as a single entity. As Van Belle points out[59] "Hence, the speed of light c emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = r^2\omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will now refer to as *the rest energy oscillation*."

The above is the main concept in Van Belle's interpretation of zitter motion of electrons, where one cycle of the electron in its zitter oscillation packs the electron's energy ($E = mc^2$) and Planck's quantum of action ($S = h$). When this 2D oscillatory motion we call electron is moving in some direction, it no longer depicts a circular but a helical motion where one could define a wavelength λ which will be related to the de Broglie wavelength of the electron and to Burinskii's[60,61] and Gauthier's[34] depiction of a photon. The idea of an oscillation packing some amount of physical action in motion is familiar with the Generalized Holographic approach (Section VII) were authors show that mass and related features emerge from a surface-to-volume angular momentum potential transfer.

Another interesting feature proposed by Van Belle considers the zitter electron oscillation as a 2 D classical oscillation, like two springs in 90-degree angle between each other, also oscillating 90 degrees out of phase independently so that their energies can add. Each oscillator has kinetic energy T described by a sine function and potential energy U described by cosine function. Recalling that in circular motion $v = \omega\, r$, and that the vibration frequency of a spring is given by ω = sqrt(k/m), where m is the mass of an object—in our case the electron mass m_e—attached to a massless spring such that $k = m_e\omega^2$, we then have

$$E = U + T = \frac{1}{2} m_e \omega^2 r^2 \left[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi) \right] = \frac{1}{2} m_e \omega^2 a^2 \ \ (each\ \ oscillator),$$

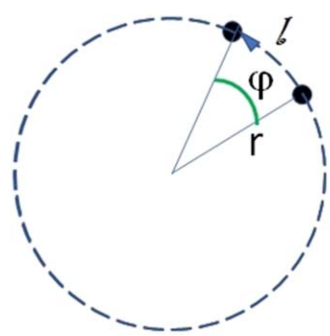

FIG. 2. (Color online) Black dot depicting circular trajectory of a particle.

where r is the turning point or amplitude of the oscillation. Evidently, this turning point is also r_C $(r = r_C)$. With two oscillators de-phased 90°, the sum gives directly

$$E = m_e r^2 \omega^2 \ \ or \ \ \frac{E}{m_e} = r^2\omega^2 = \ c^2.$$

This is interpreted as follows: when 2D oscillations occur the total energy E of the system divided by its mass m_e gives exactly the speed of light squared c^2 such that this light speed is the product of a certain oscillation amplitude of radius r and an angular frequency ω. During the rotation, if the tangential speed (which is equal to the product of the radius r of the rotation and the angular frequency ω) is the speed of light, then the mas m_e of the electron is in the oscillation itself and not in the point-like charge. In such case, $r = r_C$ effectively. Dirac's zitter tells us the velocity of the pointlike charge is equal to c. As mentioned before, if the zitter frequency is given by Planck's energy-frequency relation $(\omega = E/\hbar)$, then we can combine Einstein's $E = mc^2$ formula with the radial velocity formula $(c = r\ \omega)$, to get the zitter radius r_C which is nothing but the reduced Compton wavelength, or the Compton radius of the electron: $r_C = \hbar/m_e c = \lambda_C/2\pi \approx 0.386 \times 10^{-12}$ m. Therefore, the free electron is a pointlike charge in an electromagnetic orbital oscillation with a radius which is the Compton radius, i.e., the effective radius for interaction or interference between a photon and the electron.

We can now link Van Belle's work with Hestenes interpretation of the phase factor in the free relativistic electron wave function. Two oscillators combined is equivalent to using a linear combination of sine and cosine functions, which is Euler's function. To have the values for a general 2 D oscillator, one would have to scale Euler's unitary radius to the radius of the physical motion being the zitter radius r_c of the electron. Considering the phase factor as Euler's function depicting the real electron spin, the sign and argument of the phase factor is then not arbitrary, but representative of the spin and direction of the electron.

A third peculiar feature of Van Belle's approach is the idea on how to calculate the magnetic moment. In the preliminaries of his calculations, with no adjusting parameters, he reaches 75% of the magnetic moment value using the circular orbit. He discusses the possibility of the anomaly coming from having used circular instead of an elliptical orbit. Just as Consa,[10] Van Belle adjudicates the anomaly to a form factor.[62] A form factor would imply that there is some structure for the electron beyond the point-particle depiction.

As Van Belle points out, since an orbit has an acceleration, some force prevents the charge from going straight. If the electron with no mechanical mass, a "point charge" orbits at the speed of light around a center (its Compton radius r_C being the radius of the circular motion) the question is … why does it stays in orbit? Where does the force that keeps it in orbit come from? Van Belle proposes the answers of a 2D speed of light oscillation that could serve as a 2D "spring like" force, providing at the same time a "form factor" for the electron beyond the point-like charge model. But then again, a question remains … what is the source of such force? A very interesting issue related to this inquiry concerns the perpetual zitter motion. It is clearly possible in the case of a superconducting material, but that would imply that the electron in free space to be immersed in a superconducting medium; vacuum itself should have superconducting properties or some other equivalent characteristics. Many studies, have proposed space as a superfluid composed of bosonic fundamental particles (like the Higgs field) or Bose-Einstein Condensate,[63] which could explain perpetual motion and the origin of the force keeping the electron in orbit. These questions are addressed in Sections II D–II F.

D. The Ring and the Helical-Solenoid electron model

Many authors coincide that a realistic or deterministic interpretation of quantum mechanics arises if the electron nature is modelled beyond the probability distribution function, and the zitter motion is a perfect candidate for such an endeavor. The description of this real oscillation in time has birthed many models, from basic pictures like the initial orbiting-on-its-axis charge at light speed, up to the ring and helical-solenoid models and others developed after. These models propose that zitter is a real motion, depicted in most cases as a real orbiting charge or a real vibration such as the one presented in Section II C through Van Belle's model.

An orbiting charge creates a magnetic field. As Oliver Consa, from the department of Physics and Nuclear Engineering at UPC, recalls,[10] the idea of the electron as an oscillation was already in mind through works of André Ampere, Carl Gauss, Michael Faraday, and James Maxwell, much before quantum theory and the Copenhagen interpretation became the leading authority of the atomic arena. The zitter motion as predicted by Dirac's equation came much after, and many could consider it as a confirmation of that initial image from Ampere's 1823 tiny magnetic loops of charge,[64] up to the magneton proposed by Alfred Parson in 1915.[65] Parson had proposed a new theory in which the electron had a ring-shaped geometry and a unitary charge circulating in the ring, causing a magnetic field. Therefore, the electron would not only be the unit of electronic charge, but also of magnetic charge, or "magneton."

As recalled by Consa,[10] Parson's ring electron model postulates that the electron has a ring-shaped, extremely thin geometry that is 2000 times larger than a proton. A unitary charge flowing through the ring at the speed of light would cause an electric current and associated magnetic field. The interesting feature about this model is that it combines experimental evidence that the electron has an extremely small size (the thickness of the ring) and a relatively large size (the circumference of the ring). Such a model is equivalent to replacing the former circular trajectory in van Belle's depiction (Fig. 2) by a ring (i.e., having volume), as shown in Fig. 3.

The circumference of the ring matches the Compton wavelength λ_C while the radius corresponds to the reduced Compton wavelength $(R = r_C)$. Meanwhile, the frequency and angular frequency of the motion match the Compton frequency and reduced Compton frequency, respectively,

$$\lambda_C = 2\pi r_C = \hbar(mc).$$

In this model, the movement of the unit of charge around the ring at the speed of light produces an angular momentum of value: $L = mrv = m\,(\hbar/(mc))c = \hbar$.

By direct insertion of the electron frequency $f_e = mc^2/h$ in the Planck equation $E = hf$ we obtain Einstein's energy equation $E = mc^2$.

The moving charge generates a constant electric current, and the electric current produces a magnetic field that is induced with a magnetic moment that is equal to the Bohr magneton $\mu_e = \mu_B = e\,\hbar/2m_e$ where m_e is the electron mass. The Bohr magneton is the magnetic moment corresponding to a unitary charge that rotates with angular momentum equal to the reduced Planck constant $L = mrv = \hbar$.

As remarked by Consa,[10] a ring acts as a circular antenna, where the resonance frequency coincides with the length of the circumference. In the case of the electron ring, the resonance frequency equals the electron Compton frequency λ_C.

Consa explains that important physicists of the time supported Parson's magneton model, the most relevant of these studies was conducted by Arthur Compton,[66] who saw that the Compton Effect was best explained with Parson's ring electron model, not a spherical electron model. H. Stanley Allen compiled these studies in "The Case for a Ring Electron"[67] in 1918, where he discussed the arguments in favor of an electron in the form of a current circuit capable of producing magnetic effects. Therefore, in addition to exerting electrostatic forces, the electron would behave like a small magnet. As listed in his paper,[67] the ring electron model removes many outstanding difficulties. It was pointed out that the adoption of this hypothesis would lead naturally to the acceptance of an atomic model with a magnetic core as previously suggested by Allen.

The ring electron model was set apart in favor of quantum mechanics and Schrodinger's wave equation of the electron. We concur with Consa in his opinion; because the electron history initiated and evolved separately from that of the atom until these independent perspectives were unified into a single framework in Bohr's époque, maybe the semi-classical view was casted into shadows with the overwhelming quantum theory and a certain reject to the classical standard approach.[68]

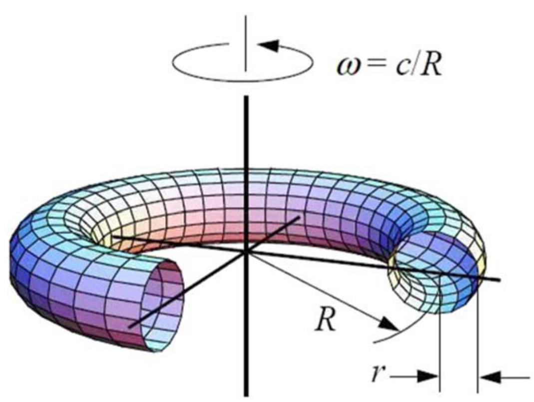

FIG. 3. (Color online) The spinning ring model of an electron. Image from Ref. 10, published under CC-BY license by IOP Publishing Ltd.

Since then, the ring electron model has been unsuccessfully revisited and authors have proposed similar ring electron models assuming that the electron is a photon trapped in a vortex, where the collision of an electron with a positron would undo the vortices and release photons.[34] However, as remarked by Consa,[10] many of these works do not adequately explain how an electric charge can be generated from a photon which in principle has no charge.

1. The Helical electron model

To address the nature of the substance that forms the ring as well as its physical characteristics and stability, Consa proposes that the entire electron charge is concentrated in a single infinitesimal point, which he calls the Center of Charge (CC), which rotates at the speed of light around a point in space that he calls the Center of Mass (CM). In his model of infinitesimal electron rotation, the ring has no substance or physical properties; it is simply the path of the CC around the CM. And since the CC has no mass, it can have an infinitesimal size without collapsing into a black hole, and it scrolls to the speed of light without violating the theory of relativity. In this view, the electron mass is distributed throughout the electromagnetic field of the electron; this mass corresponds to the kinetic and potential energy of the electron, like what Van Belle and Val Baker propose in their models.

By symmetry, the CM of the electron corresponds to the center of the ring. Thus, by replacing the geometric static ring electron model with a dynamic electron model that features a perpetual motion loop, many features of the geometric ring model become plausible because the frequency of rotation is so incredibly high that we can consider that the CC is in all points of the trajectory at the same time.

Consa shows the principles of his free electron model by making a comparison with the postulates of the Bohr atomic model: (1) The CC electron always moves at the speed of light, describing circular orbits around the CM without radiating energy. (2) The CC angular momentum equals the reduced Planck constant. (3) The electron emits and absorbs EM energy quantized according to $E = hf$. (4) The emission or absorption of energy implies an acceleration of the CM.

Since the CC moves constantly without loss of energy, we can consider the electron as a superconducting ring with a persistent current. The position of the CM of the electron at rest can be established with precision, but it is impossible to establish the position of the CC with an accuracy that is less than the radius of the ring. This feature would be equivalent to the uncertainty principle of quantum mechanics.

The electron model that Consa presents in his paper[10] and which he calls "Electron Helical Model" has much in common with the zitter models of Barut[32] and Hestenes[33] but differs in some important concepts. We mention here the most outstanding ones: Consa's helical model assumes the possibility of a substructure of the electron to explain the anomalous magnetic moment, and this helical model ignores the Dirac equation and does not assume its validity. Since

TABLE III. Spinning electron models from Ref. 10.

Geometry	$v = 0$	$v > 0$
$r = 0$	Ring	Helix
$r > 0$	Toroidal solenoid	Helical solenoid

most Zitter models start from the Dirac equation, they are relativistic. Consa's helical model obtains the Lorentz transformation equations of the theory of special relativity because of the helical motion of the electron. This last feature has tremendous implications, because then, Lorentz transformations in special relativity would emerge from geometrical considerations alone.

Consa shows that if we start from the helical motion of the electron and assume that the electron is a particle that always travels at the speed of light, it is possible to derive special relativity Lorentz transformations. He shows as well how the electron helical motion can be decomposed into two orthogonal components: a rotary motion and a translational motion.

The velocities of rotation and translation are not independent but are constrained by the tangential velocity of the electron, which should be constant and equal to the speed of light c. When the electron is at rest, the rotational velocity is equal to the speed of light, as we have discussed above. As this translational velocity increases, the rotational velocity will decrease, such that the translational velocity will never exceed the speed of light.

This last feature is particularly interesting, as it will be related to the electron model developed by Val Baker *et al.* in Section VII, where they use the Generalized Holographic Model in the context of the Bohr Hydrogen atom to derive the mass of the electron.

2. The Solenoid electron model

The magnetic moment of the electron is experimentally slightly higher than the Bohr magneton, and Consa's initial helical model could not explain the anomalous magnetic moment of the electron. He thought that the electron needed a substructure, which he describes as the solenoid electron model.

In summary, the complete model that Consa developed gathers four proposals for the spinning electron, two of which correspond to the electron at rest (the ring and the solenoid model). See Table III for more clarity. The ring model as proposed by Parson is the case where $r = 0$.

His work builds and describes in detail each case of Table III, also depicted in Fig. 4 below. The ring and helical model (cases **a** and **b**, respectively) are directly related, they concern a point-like charge where the electron has no internal structure ($r = 0$ in a and **b**). Consa shows that these two cases cannot predict the anomalous magnetic moment of the electron. To explain the anomaly, he states that the electron must possess an inner structure, which is why he describes it as a solenoid (cases **c** and **d**). The most interesting features of his helical solenoid model is that it predicts two new features for the electron: an oscillating gyromagnetic ratio, which speed is so fast that just as *zitterbewegung* (and probably related to it), it is not detectable experimentally and so we measure a mean value that we consider constant. Second,

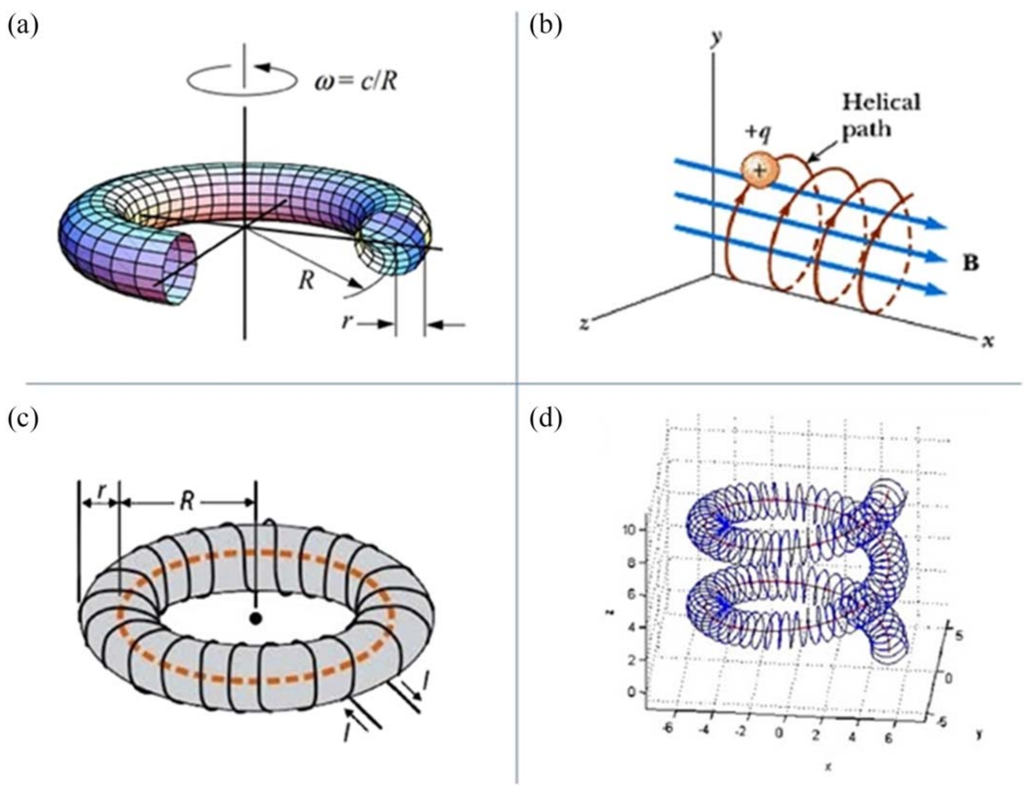

FIG. 4. (Color online) (a) Ring electron model—when r = 0. (b) Helical electron model—electron moving at v. (c) Solenoid electron model. (d) Helical solenoid electron model. Images taken from Ref. 10, published under CC-BY license by IOP Publishing Ltd.

it predicts a toroidal moment, which is a quaternal moment. Both predictions, if confirmed experimentally, would validate this model. Additionally, this model is the first theoretical model to explain the gyromagnetic ratio solely in terms of geometry, with no adjusting parameters.

Inspired by Bostick's[69] model, in which the electron takes the shape of a toroidal solenoid where the electric charge circulates at the speed of light, Consa's Toroidal Solenoid Electron Model has the electric charge as a point particle and the toroidal solenoid represents the trajectory of that point electric charge. The existence of plasmoids was discovered in 1956 by Bostick (a disciple of Compton), and it inspired both models. A plasmoid is a coherent toroidal structure made up of plasma and magnetic fields, and it is so stable that it can behave as individual objects and interact with one another. As explained by Consa, any magnetic flux is confined within the toroid, a feature which is consistent with the idea that the mass of a particle matches the electromagnetic energy contained therein. The idea of storage of electromagnetic energy in a toroidal solenoid superconductor without the loss of energy is not new, it is already known and called superconducting magnetic energy storage (SMES). Therefore, according to Consa's Toroidal Solenoid Electron model, an electron is a microscopic version of a SMES system.

The toroidal solenoid geometry is well known in electronics, where it is used to design inductors and antennas. In addition to the radius (R) of the torus, a toroidal solenoid provides two additional degrees of freedom compared to the ring geometry: the thickness of the torus (r) and the number of turns around the torus (N) with N being an integer. The toroidal solenoid is parameterized as

$$\begin{aligned} x(t) &= (R + r\cos N\omega t)\cos\omega t,\\ y(t) &= (R + r\cos N\omega t)\sin\omega t,\\ z(t) &= r\sin N\omega t, \end{aligned}$$

where the tangential velocity is

$$|r\prime(t)|^2 = (R + rcosN\omega t)^2\omega^2 + (rN\omega)^2.$$

We recall that the Compton radius r_c is associated with R ($R = r_C$). As mentioned earlier, in this model the tangential velocity is always equal to the speed of light ($|r'(t)| = c$). Following the same procedure of the helical model, the constraints are given by the conservation of translational and rotational components of the electron speed not exceeding the tangential speed (c). Then, using the Pythagorean Theorem for $R \gg r\,N$, the rotational velocity can be obtained as

$$\begin{aligned} c^2 &= (R\omega)^2 + (rN\omega)^2,\\ \frac{c}{v_r} &= \sqrt{1 + (rN/R)^2}\,. \end{aligned}$$

The second factor depends only on the geometry of the electron. Consa calls this value the helical g-factor.

If $R \gg r\,N$, the helical g-factor is slightly greater than 1

$$\mathrm{g} = \sqrt{1 + (rN/R)^2}\,.$$

As a result, the rotational velocity is dependent on the helical g-factor and slightly lower than the speed of light: $v_r = c/g$.

With this new value of the rotational velocity, the frequency f, angular frequency ω, and period T for the electron are defined by

$$\begin{aligned} f_e &= \frac{v_r}{2\pi R} = \frac{mc^2}{gh},\\ \omega_e &= 2\pi f_e = \frac{mc^2}{g\hbar}\ ,\\ T_e &= \frac{1}{f_e} = \frac{gh}{mc^2}. \end{aligned}$$

The length of a turn of the toroidal solenoid is called the arc length l. To calculate l, we need to perform the integral of the toroidal solenoid $|r'(t)|^2$ over one turn. For $R \gg r\,N$ and replacing the helical g-factor results in $l = 2\pi gR$. This means that the arc length of a toroidal solenoid is equivalent to the length of the circumference of a ring of radius $R' = gR$, or $l = 2\pi gR = 2\pi R'$.

We must take into consideration the helical g-factor as well when calculating the electron's angular momentum. The value of the rotational velocity is reduced in proportion to the equivalent radius, so that the angular momentum remains constant, just as for the ring and helical models

$$L = mR'v_r = m(gR)\left(\frac{c}{g}\right) = \hbar.$$

When calculating the angular momentum, the rotational velocity decreases in the same proportion as the equivalent radius increase, compensating for the helical g-factor. However, in the calculation of magnetic moment, the rotational velocity decreases by a factor of g, while the equivalent radius increases by a factor approximately equal to g squared. Consa states[10] that this fact is the cause of the electron's anomalous magnetic moment.

The g-factor depends on three parameters (R, r and N) but we don't know the value of two of them. Consa estimates the value of the helical g-factor using this expansion series

$$\sqrt{1 + (a)^2} = 1 + \left(\frac{1}{2}\right)a^2 + \cdots$$

Such that

$$\sqrt{1 + (rN/R)^2}\ = 1 + \left(\frac{1}{2}\right)\left(\frac{rN}{R}\right)^2 + \cdots$$

Since QED also calculates the g-factor by an expansion series where the first term is 1 and the second term is the Schwinger factor: g-factor (QED) $= 1 + \alpha/2\pi + \ldots$ then the results of the two series are very similar, and one could equate the second term of the helical g-factor series to the Schwinger factor to obtain the relationship between the radius of the torus and the thickness of the torus, such that if $(1/2)(rN/R)^2\ = \alpha/2\pi$, then

$$\frac{rN}{R} = \sqrt{\frac{\alpha}{\pi}}$$

what gives a value of helical g-factor of $\sqrt{1+(\alpha/\pi)} = 1.001\,160\,7$.

This result is consistent with the Schwinger factor, giving a value much closer to the experimental one. But more importantly, this is the first principle calculation of the g factor, which to our knowledge, is the first of such type.

Regarding the toroidal moment, it was experimentally measured in the nuclei of Cesium-133 and Ytterbium-174 in 1997.[70] In an electrostatic field, all charge distributions and currents are represented by a multipolar expansion using only electric and magnetic multipoles. In an electrodynamic field, new terms appear in the multipolar expansion which correspond to a third family of multipoles: the toroid moments. The toroidal moment can be understood as the momentum generated by a distribution of magnetic moments. The simplest case is the toroidal moment generated by an electric current in a toroidal solenoid, and the toroidal lower order term is the toroidal dipole moment, depicted in Fig. 5.

Consa presents the following argument to sustain the case of the toroidal field. Ho and Scherrer[71] hypothesized in 2013 that dark matter is formed by neutral subatomic particles and these particles of cold dark matter interact with ordinary matter only through an anapole electromagnetic moment, like the toroidal magnetic moment described above. These particles called Majorana fermions, cannot have any other electromagnetic moment apart from the toroid moment. Therefore, Consa points out that the model for these subatomic particles of dark matter is compatible with the Helical-Solenoid Electron Model. According to the Helical-Solenoid Electron Model [Fig. 4(d)] Consa evaluated the electron's theoretical toroidal moment and obtained $T \approx 10^{-40}$ Am^3. For the neutron and for the proton it should be one million times smaller. Since QM does not predict the existence of any toroidal moments, if the existence of a toroidal moment for the electron (and for any other subatomic particle) was validated experimentally, then this would validate Consa's model as well.

When the proton puzzle was at vogue, Consa identified as well[10] that if we multiply the reduced Compton wavelength of the proton—of 0.2103 fm—by four, we obtain the value of 0.8412 fm, value that corresponds nicely with the currently accepted charge radius of the proton. This supports the idea that the proton's radius is directly related to its reduced Compton radius, in agreement with Haramein's results in Ref. 104 for the proton and which he predicted before the former value of 0.8768 fm was challenged by the 2010 experiments.[105] This also supports the idea that Consa's Helical Solenoid Electron model is also valid for the proton.

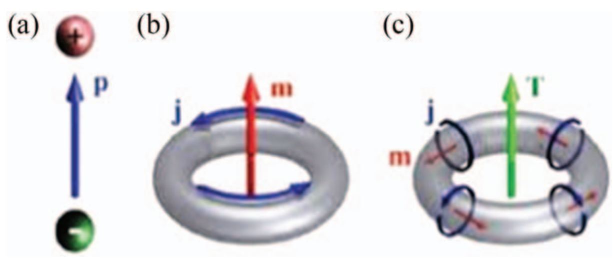

FIG. 5. Electric, Magnetic and Toroidal dipole moments. Image from Ref. 10, published under CC-BY license by IOP Publishing Ltd.

Additionally, Consa's model depicts the electron as a resonant LC circuit, where the Compton frequency is obtained as a resonance frequency, and related parameters such as the quantum Hall resistance and the quantum magnetic flux appear. The motion of the electron follows a helical path with a radius equal to the Compton wavelength λ_C and a helical pitch inversely proportional to the de Broglie wavelength. Analyzing the helical motion of the electron he obtains the same results as the theory of special relativity. This model implies that de Broglie's hypothesis about the wavelength of the electron is incorrect, raising doubts about the validity of the Schrodinger and Dirac equations, or at least of their interpretation.

Following a similar reasoning of the helical model proposed by Consa, R. Gauthier proposes a double helical photon model from which electron and positron pair production originates after interaction with a nucleus. Gauthier shows how a double-helix photon model transforming into a quantum vortex electron and positron model during electron-positron pair production can give rise to the electron's zitter frequency and energy structure. His model is explained hereafter.

E. The superluminal quantum model of the electron and positron

Inspired by de Broglie's idea of a composite photon[72–74] and considering that the production of electron-positron pairs from single photons was first observed by Blackett[75] in 1933, Richard Gauthier (Chemistry and Physics department, Santa Rosa Junior College) proposed a charged electric-dipole double-helix photon model which could originate the electron-positron pair production after interaction with a nucleus.[34,76] Although the idea is explored by other authors, in this review we will focus mainly on Gauthier's model as it gathers all main characteristics of the Dirac relativistic electron model, providing also a visualizable internal quantum trajectory structure.

The superluminal double-helix photon model proposed by Gauthier evolved from an initial superluminal quantum model of the electron and photon composed of a circulating superluminal photon-like object; an uncharged point-like quantum that moves along an open helical trajectory having a radius R which depends on its wavelength λ. One turn of the helical trajectory corresponds to one wavelength λ of the photon model.

Gauthier proposed a superluminal uncharged single-helix spin-1 photon model and a superluminal charged single-helix, internally double-looping spin-$1/2$ resting electron model. He extended the resting electron model to become a relativistic electron model composed of a helically moving spin-$1/2$ charged photon that generates the electron's de Broglie wavelength.[67] When the author learned that de

Broglie[72–74] had previously hypothesized a photon as composed of two spin-$\frac{1}{2}$ half-photons, he changed his own terminology for the "spin-$\frac{1}{2}$ charged photon"[77] composing an electron. Gauthier's spin-$\frac{1}{2}$ charged photon is now a "spin-$\frac{1}{2}$ charged half-photon". His double-helix photon is now composed of two superluminal oppositely charged spin-$\frac{1}{2}$ half-photons.

As remarked by Gauthier, Williamson and van der Mark[78] had noticed as well that the electron's spin could be explained as a double-looped circulation of a photon with wavelength equal to the electron's Compton wavelength ($\lambda_C = h/m_e c = 2.43 \times 10^{-12}$ m). Gauthier also remarks that since the Compton wavelength is the wavelength of a photon having the same energy as that of a resting electron, then the Compton-wavelength photons' double looping circulation would give a "resting" electron a zitterbewegung frequency of $\omega_{zitter} = 2\, m_e c^2/h = 2.47 \times 10^{20}$ Hz, equivalent to 1.55×10^{21} rad/sec when using ħ instead.

Gauthier proposes that a photon is composed of two superluminal oppositely charged spin-$\frac{1}{2}$ half-photons; a double helix photon model, where each helix is a circulating superluminal energy quantum. These two helices of the double-helix photon model are proposed to act together as a single quantum object—the photon—therefore, they are quantum mechanically entangled. The electron-positron pair produced when the photon passes near an atomic nucleus, is also quantum-mechanically entangled. Gauthier predicts that close experimental examination of the photon during the process of electron-positron pair production should show two opposite entangled charges of same magnitude, and he suggests experiments for such observation.

The two oppositely charged superluminal energy quanta representing the two spin $\frac{1}{2}$ charged half-photons that compose a spin 1 photon, are transverse to each other and move together in a double helical trajectory, as depicted in Fig. 6. The helical trajectory of the superluminal energy quantum's position and momentum components for the first and the second half-photon is given by the parametric equations (2) and (3), respectively, where the x and p_x and y and p_y components of Eq. (3) were equated to the negative values of the x and p_x and the y and p_y components of Eq. (2). The z and p_z components for both sets of equations are the same.

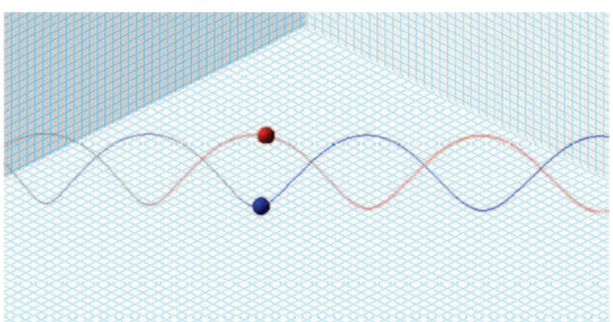

FIG. 6. The two superluminal point-like energy quanta of the superluminal double-helix model of the photon, moving on 45° helical trajectories at a speed $c\sqrt{2}$, separated by a distance $D = \lambda/\pi$ (λ is the wavelength of the photon). Each superluminal quantum composes a spin-$\frac{1}{2}$ half-photon. Figure from Ref. 34 published under CC-BY license by IOP Publishing Ltd.

Expressing the two superluminal energy quantum's coordinates of the photon model in terms of the photon's wavelength λ and angular velocity ω, we get the parametric coordinates representing the helical motion of the two helices of the double-helix spin +1 photon model

$$\begin{aligned} x_1(t) &= \frac{\lambda}{2\pi}\cos(\omega t) \qquad p_{x1}(t) = -\frac{h}{2\lambda}\sin(\omega t) \\ y_1(t) &= \frac{\lambda}{2\pi}\sin(\omega t) \qquad p_{y1}(t) = \frac{h}{2\lambda}\cos(\omega t) \\ z_1(t) &= ct \qquad\qquad\quad\; p_{z1}(t) = \frac{h}{2\lambda} \end{aligned} \tag{2}$$

and

$$\begin{aligned} x_2(t) &= -\frac{\lambda}{2\pi}\cos(\omega t) \qquad p_{x2}(t) = \frac{h}{2\lambda}\sin(\omega t) \\ y_2(t) &= -\frac{\lambda}{2\pi}\sin(\omega t) \qquad p_{y2}(t) = -\frac{h}{2\lambda}\cos(\omega t) \\ z_1(t) &= ct \qquad\qquad\quad\; p_{z2}(t) = \frac{h}{2\lambda} \end{aligned} \tag{3}$$

By reversing the signs of the y and p_y components in both Eqs. (2) and (3), we obtain the parametric equations for the spin $S = -1\hbar$ double-helix photon, each with helical radius $R = \lambda/2\pi$.

The distance D between the two superluminal quanta as they move helically opposite to each other is obtained from these last equations, giving $D = 2R = \lambda/\pi$. and this is the double-helix photon's helical diameter.

On the other hand, by differentiating the position components in Eq. (2), we obtain the speed $v(t)$ of each superluminal energy quantum in the double-helix photon model

$$\begin{aligned} v_x(t) &= \frac{dx(t)}{dt} = -\frac{\lambda\omega}{2\pi}\sin(\omega t) = -c\sin(\omega t) \\ v_y(t) &= \frac{dy(t)}{dt} = -\frac{\lambda\omega}{2\pi}\cos(\omega t) = c\sin(\omega t) \end{aligned}, \tag{4}$$

$$\begin{aligned} v_z(t) &= \frac{dz(t)}{dt} = c \\ v(t)^2 &= v_x(t)^2 + v_y(t)^2 + v_z(t)^2 \\ &= [-c\sin(\omega t)]^2 + [c\cos(\omega t)]^2 + c^2 \cdot \\ &= c^2[\sin(\omega t) + \cos(\omega t) + 1] \\ &= 2c^2 \end{aligned} \tag{5}$$

As we see, the speed of each superluminal energy quantum is $v(t) = \sqrt{c^2 2} = c\sqrt{2}$, hence the term *superluminal*. The calculation includes deriving the total momentum, obtaining the correct experimental value for the photon momentum, and the forward helical angle of the double-helix photon model, which renders 45°.

Gauthier finds that the spin z-component is the experimental value for the spin of a photon, $\hbar = h/2\pi$, while the x and y components are zero. If the double helix has the opposite helical direction from the one described by the former equations (2) and (3), then the z-component gives – ħ.

In his model, the electric charge Q of each quantum in the double helix photon model gives $Q = \sqrt{2}\; e_l$ amounting to $Q = \sqrt{2/\alpha}\ e = 16.6\ e$, where e_l is the Planck charge and e the electric charge. And the magnetic force between two moving charged point-particles whose velocities are perpendicular to each other, is zero. Therefore, the total force on each helically circulating superluminal charged particle is due to the Coulomb attractive force of the other particle, exclusively.

This 3D double-helical model of the photon can be transformed into a 3D closed-helical model of the electron and the positron during electron-positron pair production, were the amplitude and frequency parameters of the double-helix photon model equal the corresponding amplitude and frequency parameters of the electron and positron models during the proposed transformation process.

In this transformation process the incoming photon is a double-helix composite structure of two mutually circulating oppositely charged single helix half-photons that separate during electron-positron pair production and curl up their trajectories because when separated, their trajectories are no longer stable, and to stabilize they become a quantum vortex electron and a quantum vortex positron pair, as depicted in Fig. 7.

As remarked by Gauthier, if a photon of enough energy (greater than 1.022 MeV, which corresponds to the sum of the mass of an electron and a positron, or $E = 2m_ec^2$) passes near an atomic nucleus, it will produce the electron-positron pairs. He explains that the electric field of the nucleus causes the two spin-½ charged half-photons to reduce their electric charge from $\pm 16.6e$ to $\pm 1e$. which are no longer large enough to attract each other sufficiently to maintain their double-helical trajectory. The two spin-½ charged half-photons separate and their curl up separately to form an electron and a positron, now with charges e and $-e$. When curling up, each spin-½ charged half-photons each gain the electron's invariant mass m of 0.511 MeV/c^2. When travelling together as the composite photon, they did not have that mass.

The resulting trajectory of each particle, the non-relativistic electron or positron of a spin $+½\ \hbar$ moving with speed v, has the parametric equations for the x, y, and z-coordinates given in Eq. (6), expressing a circulating

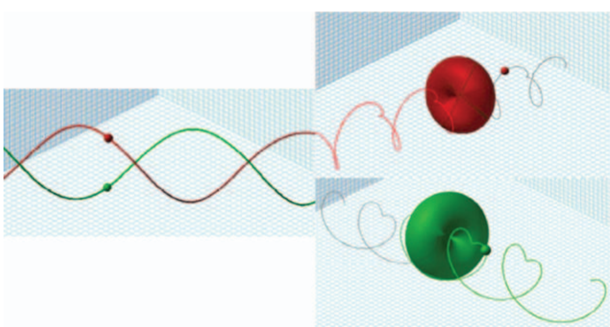

FIG. 7. When passing near an atomic nucleus (not shown) the two superluminal energy quanta forming charged spin-½ half-photons in the double-helix photon model separate, and each half photon's helical trajectory curls up forming an internally superluminal quantum-vortex electron and positron while moving away from their region of formation. Gauthier posits that the superluminal energy quantum travels on the surface of a moving mathematical torus having the electron's velocity. Figure from Ref. 34 published under CC-BY license by IOP Publishing Ltd.

superluminal energy quantum which forms a superluminal quantum-vortex model. Therefore, these equations are relativistic

$$\begin{aligned} x(t) &= \frac{\lambda_C}{4\pi}(1+\cos(\omega_{zitt}t)(\cos(\omega_{zitt}t)) \\ y(t) &= \frac{\lambda_C}{4\pi}(1+\cos(\omega_{zitt}t)(\sin(\omega_{zitt}t)) \\ z(t) &= \frac{\lambda_C}{4\pi}(\sin(\omega_{zitt}t)+vt). \end{aligned} \tag{6}$$

Here, $\frac{\lambda_C}{4\pi} = \frac{\hbar}{2mc} = 1.93x10^{-13}$m is the radius of the circular axis of the resting quantum vortex electron model and the electron's zitter angular velocity is $\omega_{zitt} = 2m_ec^2/\hbar = 1.55 \times 10^{21}$ rad/sec.

These parametric equations for a spin $+½\ \hbar$ (spin-up) electron become Eq. (7) when the electron's speed v is zero, i.e., they are non-relativistic equations

$$\begin{aligned} x(t) &= \frac{\lambda_C}{4\pi}(1+\cos(\omega_{zitt}t)(\cos(\omega_{zitt}t)) \\ y(t) &= \frac{\lambda_C}{4\pi}(1+\cos(\omega_{zitt}t)(\sin(\omega_{zitt}t)) \\ z(t) &= \frac{\lambda_C}{4\pi}(\sin(\omega_{zitt}t)). \end{aligned} \tag{7}$$

The spin $-½\ \hbar$ (spin-down) quantum-vortex electron and positron parametric equations are obtained by reversing the sign of the $y(t)$ components in Eqs. (6) and (7), respectively.

Gauthier calls this electron model "the quantum vortex model of the electron." The circulating superluminal energy quantum for the electron has a point-like electric charge of −1e, while for a positron the electric charge is +1e. The superluminal energy quantum in a resting quantum-vortex electron circulates with angular frequency ω_{zitt} in a closed helical trajectory whose circular helical axis has a circumference $C = \lambda_C/2$ and radius $R_0 = \lambda_C/4\pi$. The superluminal energy quantum in the quantum-vortex model moves along the surface of a horn torus of helical radius $R_0 = \lambda_C/4\pi$.

By means of Eq. (7), Gauthier also calculates the speed of the superluminal energy quantum when it crosses the outer equator of the horn torus, which is the maximum speed $c\sqrt{5}$. And when it passes through the exact center of the horn torus (see Fig. 8), it has got the minimum speed to be $V_{MIN} = c$ (the electron's translational speed v is zero). The double-helix photon travels forward with a velocity c, while having a constant internal superluminal speed $c\sqrt{2} = 1.414c$.

The theory provides all features calculated in detail for the superluminal photon model: spin components, forward helical angle, electric charge on each superluminal energy quantum in the double-helix photon, total momentum which gives the experimental value of a photon's momentum h/λ, magnetic moment, relativistic electron mass.[34]

Gauthier finds that the two electric charges Q and $-Q$ on the circulating superluminal energy quanta $Q = e\sqrt{2/\alpha} = 16.6$ e, were related to the electron's charge e by the fine structure constant ($\alpha = 1/137.04$). Being α a measure of the strength of interaction between an electron and a photon,

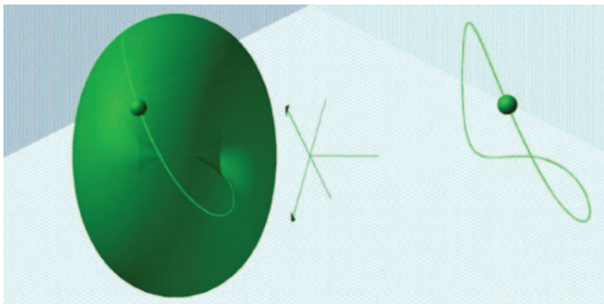

FIG. 8. Gauthier's depiction of the superluminal half-photon quantum-vortex resting electron model, formed from a superluminal spin-½ charged half-photon model, where the quantum moves on the surface of a horn torus with maximum speed $V_{MAX} = c\sqrt{5}$ and minimum speed $V_{MIN} = c$. The thickness of the horn torus corresponding to the resting electron model above is found from Eq. (7) to be $\frac{\lambda_C}{2\pi} = \frac{\hbar}{mc} = 3.86 \times 10^{-13}$ m, being its diameter twice this value: 7.72×10^{-13} m. Figure from Ref. 34, published under CC-BY license by IOP Publishing Ltd.

Gauthier suggests that this result can lead to a better understanding of the photon and/or QED.

We find as well very interesting the discussion and calculations regarding the relativistic features of the Gauthier's composed photon, presented in Ref. 34.

F. The zitterbewegung electron and Occam's Razor

Inspired mostly by Hestenes, Giorgio Vasallo (Dipartimento di Ingegneria dell'Università degli Studi di Palermo) and collaborators introduced a model where concepts like mass, energy, time, and information are directly related.[11,35] In this work, Vasallo cites the definition that B. Sidharth gives to the *zitterbewegung*,[35] "The well-known *Zitterbewegung* may be looked upon as a circular motion about the direction of the electron spin with radius equal to the Compton wavelength (divided by 2π) of the electron. The intrinsic spin of the electron may be looked upon as the orbital angular momentum of this motion. The current produced by the *Zitterbewegung* is seen to give rise to the intrinsic magnetic moment of the electron."[79] As he remarks, Hestenes considers the complex phase of the wave function solution of the traditional Dirac equation as the phase of the zitter rotation, showing "the inseparable connection between quantum mechanical phase and spin" consequently rejecting the "conventional wisdom that phase is an essential feature of quantum mechanics, while spin is a mere detail that can often be ignored."[80]

As Vasallo explicitly says,[35] the aim of his papers is to "present a gentle introduction to an electron Zitterbewegung model together with some observations that deems to reinforce its plausibility."

At the time Vassallo *et al.* (and many authors in this review) derived his model, measurements were mostly based on an international system SI which was not entirely related to fundamental constants such as speed of light c and the Planck's quantum $\hbar$. Therefore, they were dependent upon human convention. Though this is no longer the case, Vassallo had very well identified and anticipated this limitation[11] "Considering that a measure is an event localized in space and time, the quantum of action can be seen, in some cases, as an objective entity in some respects analogous to a bit of information located in the space-time continuum. In accordance with Heisenberg's uncertainty principle, the result of the measurement of some values (such as angular momentum) cannot have an accuracy less than half a single Planck's quantum. Therefore, to simplify the interpretation of physical quantities, it may be useful to adopt a system in which both the speed of light and the quantum of action are dimensionless quantities (pure numbers) having a unit value, i.e., $c = 1$ and $\hbar = 1$. In this system, the constancy of light speed makes it possible to use a single measurement unit for space and time, simplifying, in many cases, the conceptual interpretation of physical quantities. The energy of a photon, a 'particle of light,' is equal to Planck's quantum multiplied by the photon angular frequency."

Keeping this in mind and recalling from Section I C that T represents the period of a single complete oscillation and λ the relative wavelength, it is possible to write

$$E = \hbar\omega = \frac{2\pi\hbar}{T} = \frac{2\pi\hbar c}{\lambda}$$

being $\hbar$ the proportionality factor of the smallest increment of energy E versus the frequency ω of an electromagnetic wave.

By using natural units, period and wavelength coincide, simplifying the above equation to

$$\left[E = \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{\lambda}\right]_{NU}.$$

The equation above shows the relation between fundamental concepts such as space, time, energy, and mass, giving the possibility to express an energy value simply as a frequency or as the inverse of a time, or even as the inverse of a length. Equivalently, it allows us to use as a measurement unit of both space and time a value equal to the inverse of a particular energy value as the electron-volt (eV).

Therefore, to compute photon wavelength in vacuum with natural units it suffices to divide the constant 2π by its energy. This value will correspond exactly to the period of a complete oscillation. Hence, in natural units the inverse of an eV can be used as a measurement unit for space and time

$$\begin{aligned} L_{(1eV)} &= 1\ \mathrm{eV}^{-1} \approx 1.9732705 \times 10^{-7}\mathrm{m} \approx 0.2\,\mu\mathrm{m} \\ T_{(1eV)} &= 1\ \mathrm{eV}^{-1} \approx 6.582122 \times 10^{-16}\mathrm{s} \approx 0.66\,\mathrm{fs} \end{aligned}.$$

In such case, an angular frequency can be measured in electron volts

$$1eV \approx 1.519\,268 \times 10^{15}\,\mathrm{rad\,s^{-1}}.$$

Vassallo remarks that following these concepts, it is possible to define a direct connection between fundamental concepts such as information, space, time, frequency, and energy. For instance, a single photon will carry a quantum of information that will have a "necessary reading time" and a spatial dimension which are inversely proportional to its energy. Vassallo gives the example of the radio antennas, where their length is proportional to the received or transmitted "radio photons" wavelength and inversely proportional to their frequency (energy) and to the number of bits that can be exchanged in a unit of time. Under this view, the concept of energy is closely linked to the "density" of information in space and in time. As we will see in Section VII, this too is also related to the generalized holographic approach of Haramein *et al*.

Einstein's $E = mc^2$ expressed in natural units becomes particularly explanatory, as Vassallo remarks. If $[E = m]_{NU}$; then mass is energy so a particle having a given mass has a precise amount of energy associated with it. The deep connection between the concepts of space, time, frequency, and energy, associate the electron rest mass m_e to an angular frequency ω_e, a length r_C and a time T_e; constants that acquire a clear meaning if one adopts the electron model consisting of a current ring generated by a massless charge that rotates at the speed of light, along a circumference with radius equal to the electron reduced Compton wavelength, $r_C = {}^{\lambda_C}\!/_{2\pi} \approx 3.861593 \times 10^{-13}$m. Note that in the original work from Vassallo, notation for Compton radius is r_e instead of r_C and classical electron radius is r_0 instead of r_e.

Einstein's formula can be expressed as

$$E_e = \ m_e c^2 = \hbar \ \omega_e = \frac{\hbar c}{r_C} = \frac{\mathrm{h}}{T_e}.$$

From which the mass of the electron m_e can be obtained as[11,35]

$$m_e = \frac{\hbar \ \omega_e}{c^2} = \frac{\hbar}{c r_C} = \frac{\mathrm{h}}{c \lambda_C} \approx 9.109383 \times 10^{-31} \ \mathrm{kg}.$$

Or, in natural units $[NU]$,

$$\left[E_e = \ m_e = \omega_e = \frac{1}{r_C} = \frac{2\pi}{T_e} = 0.511 \times 10^6 \ \mathrm{eV} \right]_{NU}.$$

According to this model, the charge is not a point-like entity, it is distributed on a spherical surface representing the elementary charge e as shown in Fig. 9, where ω_e is the angular frequency of the rotating charge, T_e is its period, and r_C is its orbit radius. The elementary charge finite dimension imposes a constraint: all points of the surface of the spinning charged sphere must have the same instantaneous speed of light c and the same angular speed. Within a frame that rotates at zitter frequency, this spinning charged sphere rotates around its center with opposite speed with respect to the zitter angular frequency $\omega_0 = -\omega_e$, as shown in Fig. 9.

The relationship of angular and tangential velocities from Vassallo's work is in close relation with the succession of angular velocity relationships in Daywit's Planck particle model (Section IV) and in Val Baker's holographic model for the electron mass (Section VII), and that also relate directly to the hierarchy problem.

As seen in Fig. 9, Vassallo's electron model is not point-like; in (a) the charge rotates with angular speed ω_0 around the axis passing through the center of the sphere and, therefore, all points of the sphere have the same absolute speed c. Interestingly, in this model, the charge describes a torus with cross section equal to πr_e^2 and volume $2\pi^2 r_C r_e^2$ during the rotation around the origin C.

The current loop is associated with a quantized magnetic flux Φ_M equal to Planck's constant ($h = 2\pi \ \hbar$) divided by the elementary charge e as $\Phi_M = h/e$, which in natural units becomes: $[\Phi_M = 2\pi/e]_{\mathrm{NU}}$.

The current loop generated by the elementary rotating charge generates a centripetal Lorentz force due to the magnetic field associated with the rotation. In natural units, the value of this elementary charge equals the square root of the ratio between the charge radius r_e and the orbit radius r_C,

$$\left[e = \sqrt{\frac{r_e}{r_C}} = \sqrt{\alpha} \approx 0.0854245 \right]_{NU}.$$

As remarked by Vassallo, it is very important to note that the ratio r_C/r_e is exactly equal to the inverse of the fine structure constant,

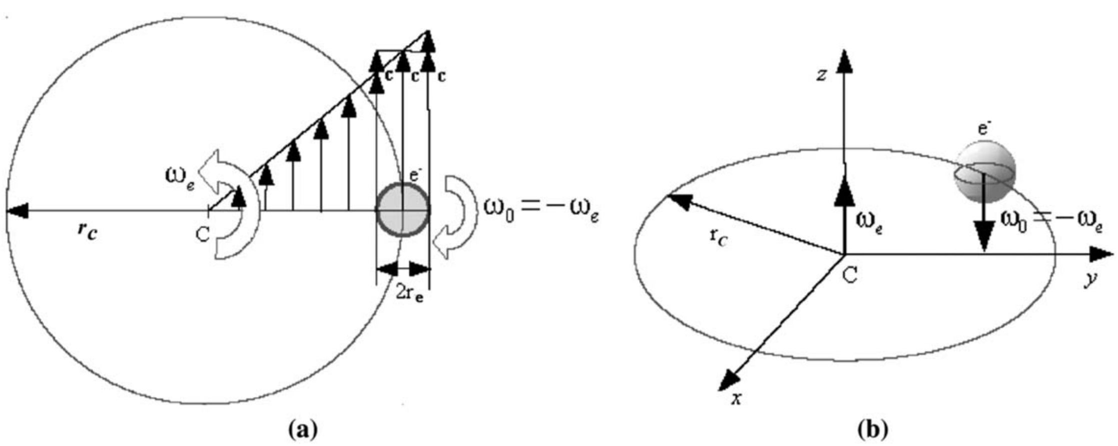

FIG. 9. (a) ZBW model and speed diagrams of the electron charge (e−). All points of the sphere have an absolute speed equal to c. (b) 3D representation. The charged sphere is rotating with the relative angular speed $\omega_0 = -\omega_e$ on the trajectory having radius r_C around the vertical axis passing through the center of the sphere. Image from Ref. 35, reprinted with permission from J. Cond. Mat. Nucl. Sci. **25**, 76 (2017). Copyright 2017 International Society for Condensed Matter Nuclear Science.

$$\frac{r_C}{r_e} = \alpha^{-1} \approx 137.035999.$$

The rotating charge is also characterized by a purely electromagnetic momentum p,

$$p_c = eA = e\frac{\Phi_M}{2\pi r_C} = \frac{\hbar\omega_e}{c} = \frac{\hbar}{r_C} = m_e c,$$

where we can recognize the variable $A = \frac{\hbar}{e\ r_C}$ indicating the vector potential seen by the rotating charge. If we multiply this charge momentum p_c by the radius r_C, we obtain the intrinsic angular momentum $\hbar$ of the electron: $p_c\, r_C = \hbar$.

In NU, the momentum p_c has the dimension of energy, becoming exactly the electron mass–energy at rest m_e,

$$\left[p_c = eA = \mathrm{E}_e = \frac{1}{r_C} = m_e = \omega_e\right]_{NU}.$$

Concerning the magnetic Aharonov–Bohm effect, it is described by a quantum law that gives the phase variation ϕ of the "electron wave function" starting from the integral of the vector potential $\mathrm{A}_\triangle$ along a path as[11]

$$\phi = \frac{e}{\hbar}\int A_\triangle .\ dl.$$

In this zitter model, the electron wave function phase is the charge rotation phase, connecting as well with Hestenes work. Since vectors $\mathrm{A}_\triangle$ and dl have the same tangent direction to the trajectory of the elementary charge, Vassallo's work proposes a possible test which consists in verifying that the phase shift ϕ along the circumference of the zitter orbit is equal exactly to 2π radians, using the above equation. This gives

$$\phi = \frac{e}{\hbar}\int A_\triangle .\ dl = \frac{e}{\hbar}\int_0^{2\pi r_C} A\ dl = \frac{e}{\hbar}\int_0^{2\pi r_C} \frac{\hbar}{e\ r_C}$$
$$dl = \frac{e}{\hbar}\ \frac{\hbar}{e\ r_C} 2\pi r_C = 2\pi.$$

Authors find the same consistency when calculating the electric Aharonov–Bohm effect. They show that consistency goes further, proving the Proca and electromagnetic Klein-Gordon equations, the electromagnetic Dirac equations, and so on.[35] Just as Consa,[10] Vassallo *et al*. additionally find the inductance L_e and capacitance C_e of the electron.

In particular, the section concerning the geometric interpretation of the relativistic electron mass and de Broglie wavelength in Vassallo's work,[11] is very compelling. We will include it here after. An electron moving along an orthogonal axis z to its charge rotation plane will describe a helical trajectory (as mentioned also in Section I C) with length $D = c\ \Delta t$ and z-axis length $d = v_z\ \Delta t$. Additionally, in NU the electron mass is exactly equal to the inverse of the helix radius r: $m = r^{-1}$. Therefore, an acceleration along z implies a smaller radius and, hence, a mass increase.

It is possible to write the value of the radius r and its related mass variation as a function of v_z utilizing the Pythagorean theorem,

$$r = r_C\sqrt{1 - \frac{v_z^2}{c^2}},$$
$$m = \frac{\hbar\omega}{c^2} = \frac{m_e}{\sqrt{1 - \frac{v_z^2}{c^2}}}.$$

While the charge momentum is proportional to the angular frequency and it has a direction tangent to the helical path, such that the relativistic momentum of charge is

$$\mathrm{p_c} = \mathrm{eA} = \hbar\omega/\mathrm{c} = \hbar r,$$

Or, in natural units, $\left[p_c = \ \omega\ = \frac{1}{r} = m\right]_{NU}$.

As explained by Vassallo, this suggests the following interpretation of the Heisenberg uncertainty principle: an electron whose charge has a momentum p_c, cannot be confined within a spherical space of radius R less than r. Therefore, $R > r = \hbar/p_c$.

The charge momentum vector $p_c = \mathrm{eA}_\triangle$ can be decomposed into two components: $p_\perp$ that is orthogonal to electron velocity and $p_{||}$ that is parallel or in the z-direction such that $p_c = p_\perp + p_{||}$, then the magnitude of component $p_\perp$ is a constant, independent from velocity $\mathrm{v_z}$, and is proportional to the charge angular speed ω_e in the xy-plane. We have then

$$p_\perp = \frac{\hbar\omega_e}{c} = m_e c.$$

Or in NU: $[p_\perp = \omega_e = m_e]_{NU}$, while the component $p_{||}$ is the momentum of the electron and is proportional to the instantaneous angular frequency $\omega_z = v_z/r$ such that

$$p_{||} = \frac{\hbar\omega_z}{c} = \frac{\hbar v_z}{cr} = \frac{\hbar\omega}{c^2}v_z = mv_z,$$

which in NU gives: $\left[p_{||} = \omega_z = \frac{v_z}{r} = mv_z\right]_{NU}$.

By Pythagorean theorem, we obtain the following equations:

$$\omega_e = \frac{v_\perp}{r} = \frac{\sqrt{c^2 - v_z^2}}{r} = \frac{\sqrt{c^2 - v_z^2}}{r_C\sqrt{c^2 - \frac{v_z^2}{c^2}}} = \frac{c}{r_C},$$

which in consequence gives: $\omega = c/r$. But since $\omega_z = v_z/r$ then the sum of the angular frequencies yields the following relations:

$$\omega^2 = \omega_e^2 + \omega_z^2,\ \ p_c^2 = p_\perp^2 + p_{||}^2 \text{ and } mc^2 = m_e^2c^2 + m^2v_z^2.$$

Then, we have that $p_{||}\ = \mathrm{m}v_z$

Following the de Broglie hypothesis, ω_z is the instantaneous angular frequency associated with a particle with rest mass m_e, relativistic mass m, and velocity $v_z = \omega_z\ r$. Therefore,

$$p_{||} = mv_z = \frac{\hbar\omega}{c^2}v_z = \frac{\hbar}{cr}v_z = \frac{\hbar\omega_z}{c} = \hbar\frac{2\pi}{\lambda} = \hbar k,$$

where we can see that

$$\frac{p_{||}}{k} = p_{||}\frac{\lambda}{2\pi} = \hbar$$

and where the term $k = 2\pi/\lambda$ is the wave number of the electron and λ is the related de Broglie wavelength. This means that if we observe the electron at a spatial scale much larger than its Compton wavelength and at a time scale much higher than the very short period $T \approx 8.1 \times 10^{-21}$ s of the *Zitterbewegung* rotation period for a constant speed v_z, the electron can be approximated to a point particle, provided with "mass" and charge, which moves with a uniform motion along the z-axis of the helix, as others authors in this review have also remarked in their models.

An in NU this is:

$$\left[p_{||} \;= \mathrm{m}v_z = \omega v_z = \frac{v_z}{r} = \omega_z = \frac{2\pi}{\lambda} = k\right]_{NU}.$$

Figure 10 represents the helical trajectories of electrons moving at different speeds.

In Vassallo's model, the electron is subjected to Larmor precession in the presence of an external magnetic field, and its spin value $\pm\ \hbar/2$ is interpreted as the intrinsic angular momentum component parallel to the magnetic field. With future technology, this feature could allow to align the intrinsic angular momentum of a sufficient number of electrons to favor the formation of a coherent superconducting and superfluid condensate state.

In this state, Vassallo remarks that "the electrons would behave as particles with whole spin ħ and would no longer be subject to the Fermi–Dirac statistic. The compression effect (pinch) of an electrical discharge, accurately localized in a very small 'capillary' volume, inside which a very rapid and uniform variation of the electric potential occurs, could favor the formation of a superconducting plasma. Because of Aharonov–Bohm effect, the conjecture is based on the possibility that a rapid, collective, and simultaneous variation of the *Zitterbewegung* phase catalyzes the creation of coherent systems like those described by Puthoff and Piestrup."[81]

In summary, Vassallo's model shows that the electron characteristics may be explained by a massless charge distributed on the surface of sphere that rotates at the speed of light along a circumference with a radius equal to the reduced electron Compton wavelength ($\approx$ 0.386159 pm), a value that is two times the one proposed by Hestenes in Eq. (33) of Ref. 82.

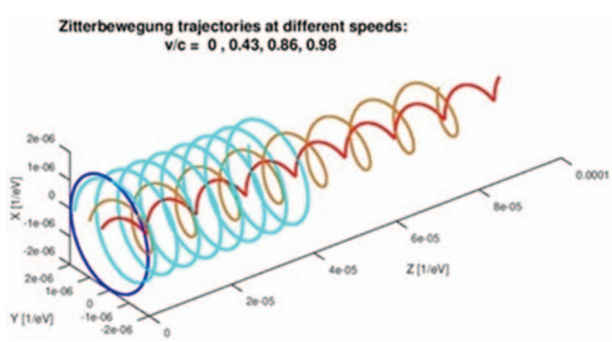

FIG. 10. Zitter frequencies at different speeds. Image from Ref. 11, reprinted with permission from J. Cond. Mat. Nucl. Sci. **29**, 525 (2019). Copyright 2019 International Society for Condensed Matter Nuclear Scienc.

He shows that the electron mass–energy, expressed in natural units, is equal to the angular speed of the zitter rotation and to the inverse of the orbit radius (i.e., $\approx$511 keV), whereas the angular momentum is equal to the reduced Planck constant. In Vassallo's model, a relativistic contraction of the zitter radius and the corresponding instantaneous zitter angular speed increases as the electron speed increases, contrary to Hestenes proposal.

This is the final model that we have explored in this section, addressing the electron model in terms of *zitterbewegung*. In Sections III–VII, we will address the electron models that are based on the vacuum fluctuations (which of course, are related to zitter as well), starting with Stochastic electrodynamics. They might serve as a foundation for the origin of zitter, since zitter is linked to the vacuum fluctuations.

III. STOCHASTIC ELECTRODYNAMICS AND INERTIAL MASS

A series of articles by Harold E. Puthoff (Institute for Advanced Studies at Austin), Alfonso Rueda (Department of Electrical Engineering, California State University) and Bernhard Haisch (Lockheed Palo Alto Research Laboratory), addressing Stochastic electrodynamics SED[37,83–89] and a new QED analysis[90] have been published since 1980, showing that a massless point-charge oscillator accelerating through the zero-point field—ZPF—will experience a Lorentz force coming from the magnetic components of the zero-point energy fluctuations or zpe that are directly proportional to acceleration, and therefore deriving the Newtonian mechanical relation $F = ma$ directly from electrodynamics. Forces would then be originated by electromagnetic quantum vacuum appearing as inertial mass, with huge implications in all of physics, since the origin of Einstein's principle of equivalence would be explained by this mass generating mechanism based on ZPF.

More precisely, they propose that inertia is an electromagnetic resistance arising from the known spectral distortion of the ZPF in accelerated frames[37] showing that electromagnetic fields can give rise to ZPF momentum flux when applying the standard relativistic transformations. An object scattering this ZPF momentum flux will produce a reaction force that can be interpreted as a contribution to the object's inertia. Under the assumption that scattering of the ZPF radiation takes place at the level of quarks and electrons, they obtain both Newton's equation of motion $F = ma$ and its relativistic covariant generalization.[86,91] Additionally, they propose that this scattering must take place at the Compton frequency of the particle. This interpretation of mass lead them directly to the de Broglie relation that characterizes the wave nature of that particle in motion, $\lambda B = h/p$, thereby connecting electrodynamics to the quantum wave nature of matter.

In other words, the $F = ma$ law of mechanics and its relativistic counterpart can be traced back to a purely electromagnetic effect, in which a law of inertia can be derived for matter comprised of electromagnetically interacting particles that originally are massless. This means that the mass-like properties of matter are manifestations of the inherent energy-momentum of the quantum vacuum radiation field, reason why authors call it the *quantum vacuum inertia hypothesis*.

In their quantum vacuum inertia hypothesis, inertial and gravitational mass are the same; inertial mass arises upon acceleration through the electromagnetic quantum vacuum, while gravitational mass—weight—is viewed as the acceleration of the electromagnetic quantum vacuum past a fixed object in a gravitational field, where the quantum vacuum radiation, associated with the free-falling frame that comoves instantaneously with the object, follows the curvature posited by general relativity.

In this SED theory, the interactions of the quantum vacuum radiation field with massless particles results in Schrodinger's zitter, i.e., electrons random light speed fluctuations, which can be shown to cause a point-like particle to appear spread out in volume over a region—the Compton sphere. Authors think of particles' rest mass to be a manifestation of the energy associated with *zitterbewegung*,

The connection between zitter and ZPF has triggered investigations for a mass generation alternative to the Higgs mechanism in the Standard Model (where all particles would be massless and interaction with the Higgs field confers their mass). The Higgs mechanism is mediated by Higgs bosons (quantum of the Higgs field) and attempts to detect it have apparently succeeded in 2013.[92] Nevertheless, as Haisch et at explain, this mechanism can only account for the electro weak sector of the standard model, representing less than 2% the total inertial mass of protons and neutrons. In the Standard Model of particles, the remainder masses would come from a different unproven mechanism regarding strong confining fields and their quantum counterpart—the gluons. As Haisch states[37] "The quark masses, the gluon fields and other strong interaction energies would not be affected by a Higgs field. The origin of inertial mass of ordinary matter is thus a wide-open question."

Just like the Casimir effect proves the vacuum can exercise a measurable force in nanoscopic and macroscopic regime,[93] which very recently included torque[94] and repulsive Casimir,[95] this zero-point field or vacuum fluctuations could provide a mechanism that keeps the charge in orbit.

Several authors regard the zero-point field as a real Planck plasma, therefore, relating it to the Planck quantum of action or spin, which then would be inherent to the vacuum structure. Among these authors, we have William Daywitt, whose Planck Vacuum theory will be addressed hereafter.

IV. THE DIRAC ELECTRON MODEL IN THE PLANCK VACUUM THEORY

Inspired by former works from Puthof[86] and Saharov,[96] William Daywitt (former researcher at National Institute for Standards and Technology), proposes the Planck Vacuum theory (PVT)[97] were the source of the quantum vacuum (QV) is the Planck vacuum (PV) described as an omnipresent degenerate state of negative-energy Planck Particles (PP) characterized by the triad $(-e_l, m_l, r_l)$ being e_l, m_l, and r_l the PP charge, mass (Planck mass), and Compton radius (Planck length), respectively. The massless bare charge e_l common to all charged elementary particles is related to the observed electronic charge e through the fine structure constant $\alpha = e^2/e_l^2$. When a massless bare charge e_l travels freely at a constant speed v following a straight line, its bare Coulomb field e_l/r^2 polarizes the Planck Vacuum and the PV responds to the perturbation by producing magnetic and Faraday fields that interact with the bare charge iteratively, leading to the relativistic electric and magnetic fields that are identified to the charge as a single entity. The force perturbing the PV is given by e_l^2/r^2, where one of the charges e_l in the product e_l^2 belongs to the free charge and the other to the individual Planck particles making up the degenerate negative energy PV. When measuring the force between two free elementary charges in the laboratory we observe e^2/r^2 $(= \alpha e_l^2/r^2)$, where e is the observed electronic charge and α is the fine structure constant.[98] The PP charges e_l move randomly around their equilibrium positions originating the zero-point vacuum fluctuations or QV.

In the case of the Dirac electron, the bare charge has a mass m that exerts an additional force to that of the polarization or electromagnetic force e_l^2/r^2, reason why the description of the response of the PV to the electron's uniform motion must include the attractive curvature or gravitational force $m_e c^2/r$ as the massive charge perturbs the PV. The radius at which the magnitudes of these two forces equal

$$\frac{m_e c^2}{r} = \frac{e_l^2}{r^2}$$

establishes the Compton radius for the electron $r = r_c$.

Daywitt finds a succession of Compton relations $r_C m_e c^2 = r_l m_l c^2 = e_l^2 = c\hbar$ that tie the electron $r_C m_e c^2$ to the Planck particles $r_l m_l c^2$ within the PV. Here the charges in the product e_l^2 are massless point charges. To address the hierarchy problem between electron and proton, we include the proton in the succession as follows. The particle-PV coupling that both electron $(-e, me)$ and proton cores $(+e, mp)$ exert on the PV state, along with their coupling constants $Fe\,(r_C) = 0$ and $Fp\,(r_{C_p}) = 0$, are

$$F_e(r) = \frac{e_l^2}{r^2} - \frac{m_e\ c^2}{r} \quad \text{and} \quad F_p(r) = \frac{e_l^2}{r^2} - \frac{m_p\ c^2}{r}$$

And the resulting Compton radius $r_C = \frac{e_l^2}{m_e\ c^2}$, $r_{C_p} = \frac{e_l^2}{m_p\ c^2}$ leads to the thread of relations

$$r_C m_e c^2 = r_{C_p} m_p c^2 = e_l^2 = r_l m_l c^2 = c\hbar \tag{8}$$

being ħ the reduced Planck constant. Propagation of electron and proton particles in free space is insured by the vanishing of $Fe\,(r_C)$ and $Fp\,(r_{C_p})$ that free the electron and proton from being tied by their coupling forces to the vacuum state.

From the Compton radius, we obtain the mass-energy of both particles $m_e c^2 = e_l^2/r_c$ and $m_p c^2 = e_l^2/r_{C_p}$ leading to

$m_p = (r_{c_p}/r_c)m_e$, where the ratio r_C/r_{C_p} is almost 1836. Since m_e is assumed structureless, this ratio suggests that the constant 1836 can be thought of as the "proton structure constant." In PVT, the so-called structure appears in the proton rest frame as a small spherical "collar" surrounding the proton core, interpretation that coincides exactly with the unified approach developed in the holographic electron model to be explained in Section VII.

The author goes further and posits that the PV is a composite state pattern following a hierarchy of Compton relations $r_C m_e c^2 = r_{C_p} m_p c^2 = r_{C_i} m_i c^2 = r_l m_l c^2 = e_l^2$ where i between the proton and Planck-particle products represent heavier or denser intermediate-particle states. One could summarize this succession as $r_l m_l c = \; r_{C_i} m_i c = \; e_l^2/c$ for a given mass m_i and Compton radius r_{Ci} of any general subatomic particle. The components of this expanded vacuum state correspond to the sub-vacuum associated with these particles, for instance, the electron-positron Dirac vacuum ($r_C m_e c^2$) in the electron case. As the energy decreases, the negative-energy states in Dirac Eq. for the free electron goes through the succession of sub-vacuum states ending its increasingly negative-energy descent at the Planck-particle stage $r_l m_l c^2$ which acts as a boundary condition instead of an unbounded or negative-energy infinity. Through this assumption the PV model would include all massive-particle quantum vacuum which corresponds to the collection of sub-vacuum states.

In PVT model, the electron mass results from a massless naked charge being driven by ultra-high-frequency photons of the zero-point electromagnetic vacuum; the electron charge exhibits a small fluctuation about its center of motion, the so called *Zitterbewegung* motion which would confer the mass just as SED theory and others collected in this review have proposed. The resulting massive-charge collisions with the active PV produce a cloud of electron-positron pairs around that charge, and then the massive free charge exhibits an exchange type of scattering with some of the electrons in the pairs that increase the free electron's apparent size during the process. Daywitt assumes that the massive-particle component of the quantum vacuum does not exist in free space except under stressful conditions.[98]

Of special interest in the analysis of the zitter motion in PVT is the estimation of the onset radius for electron-positron pair production as the Dirac electron is approached (in its rest frame). Here the author shows that the standard estimate value is taken to be the electron Compton radius, which is significantly overestimated, over four times bigger than the value obtained by Daywitt using the coupling force. When separated the Compton radius (r_C) from the onset radius ($r_C/4.5$), the electron-positron pair production falls outside the onset, and the Compton radius can no longer be associated with this pair production process.

The first Compton radius $r = r_C$ refers to the vanishing coupling-force sphere centered on the point electron (in its rest frame), and the pair-creation onset radius $r_C/4.5$ refers to the possible onset of electron-positron pairs. Consequently, the *zitter* is not related to the pair-production phenomena of an over-stressed ($V(r) \geq 2mc^2$) PV state. Instead, it seems to be the consequence of a PV resonance with the resonant frequency $2c/r_C$ associated with a corresponding r_C-sphere. As explained by Daywitt, this separation of the Compton and onset radii makes the zitter a realistic model and leads to its clear explanation in terms of vacuum dynamics. In his opinion, direct attribution of *zitterbewegung* phenomenon to the dynamics of the electron particle rather than the dynamics of the vacuum state misleads and creates confusion; the *zitterbewegung* is not a mathematical curiosity but a fundamental and crucial part of the Dirac electron theory.

Concerning the dynamics of the electron, it is summarized as follows: the zero-point fluctuations of the PP within the degenerate negative energy PV create zero-point electromagnetic fields that exist in free space. When the charge is injected into free space (presumably from the PV), the driving PV force generates the electron mass, thereby creating the point electron characterized by its bare point charge e_l, its derived mass m_e, and its Compton radius r_C. Concerning the point-charge aspect of the model, it should be recalled that experimentally, the electron appears to have no structure at least down to a radius around 10^{-20}cm, nine orders of magnitude smaller than the electron's Compton radius. Also, in this PV model the energy that the charge absorbs from the field is re-radiated back into free space leaving the isotropy and spectral density of the zero-point background unchanged, just as in the SED model.[83–90]

His picture of the Dirac electron model is the following: centered at the origin of the rest frame is the massive point charge with an effective volumetric radius $\langle r \rangle \approx 0$ surrounded by an hypothetical sphere of radius $r_C/4.5$ within which the positive energy of the free electron and the negative energy of the PV overlap (allowing electron-positron pairs to be excited) and this configuration is again surrounded by a spherical annulus of radius $r_C/4.5 < r < r_C$ where pair production does not occur. Beyond the r_C-sphere ($r \geq r_C$) is a region of diminishing PV stress, a compression that decreases with increasing r according to the force difference. Very interestingly, this picture depicts a similar situation as that of Alexander Burinskii and his Dirac-Kerr-Newman model for the electron, where the electron "particle" is also separated in three analogous regions of space, providing a sort of form factor for the electron.

Non-relativistic calculations in the PVT Bohr-hydrogen-atom model significantly expand the Bohr concept concerning the quantized angular momentum, the hydrogen energy levels E_n, the orbit radii r_n, the ratios E_n/r_n, and the hydrogen Rydberg constant RH. The Bohr quantization of the angular momentum is directly related to the electron-PV coupling force and the ratios E_n/r_n are proportional to the n-ratio from the Schwarzschild line element for Einstein field equations.[99] This last feature also connects to Alexander Burinskii's work, who posits that the huge spin of elemental particles such as electrons drag and deforms space-time as a black hole (BH) singularity would, naming his model the Dirac–Kerr–Newman electron.

Before we address Burinskii's model, we present hereafter the fully relativistic model from Wilson, based on Dirac's equation. With no further assumptions beyond the fact that the Dirac equation (DE) provides a physical description of the free electron, through his QED-P model Wilson's calculations show that the DE describes an internal structure for the electron, arriving from an unclassical deterministic approach to results in the same line than that of the semiclassical models formerly presented; an electron's point charge is in rapid oscillation about its apparent center of mass, creating a charge shell and magnetic moment over time that defines the intrinsic electron properties. He provides a plausible framework to unify the concepts of zero-point energy fluctuations and *zitterbewegung* as well.

V. THE DIRAC MODEL DEM AND QED-P

There have been several attempts to connect zitter and zpe. For instance, Sakurai conjectured that zitter emerges because of the virtual electron-positron pairs (or vacuum fluctuations) influence on the electron.[100] If virtual particles are concerned, then one could think of mechanisms where the original electron annihilates the virtual positron or fills the hole, then the virtual electron becomes the new real electron appearing at a distance from the original electron. In this spirit, inspired in Barut and Bracken,[32] Basil Davis[101] proposes that it is the charge that is transported spatially, not the mass, and associates *zitterbewegung* to this movement of charge, while the mass moves slower than c. Therefore, there would be no violation of SR, and charges instead of masses would be involved in the virtual pair creation and annihilation mechanism described above.

If that were the case, the mass and charge of the electron would need different position operators, and the Dirac electron model—abbreviated here as DEM—from James Wilson heads in such direction.

Wilson's work[15–19] is based on the original Dirac equation, which in principle concerns Dirac's fully relativistic Hamiltonian for a free particle

$$H_D = c\alpha \cdot \boldsymbol{p} + m_e \ c^2 \ \beta.$$

Since an electron's wave function is described by two independent spin base states, each with positive or negative energy, the solution of Dirac's equation is a four-component spinor and the Hamiltonian is a 4×4 matrix, where $\boldsymbol{\alpha}$ and β matrices are independent of space and time coordinates since they commute with position and momentum operators.

The velocity or time dependence of the position operator for the above equation gives $c\boldsymbol{\alpha}$ in the Heisenberg picture. This term $c\boldsymbol{\alpha}$ in the Dirac Hamiltonian is the Dirac velocity with eigenvalues of ± 1. This implies that the velocity operator has two eigenvalues, $\pm c$, meaning that the absolute value of the particle's velocity in each spatial direction is c, what defies special relativity. To solve this conundrum, it has been interpreted instead that a superposition of positive and negative energy states emerges as a solution to the Dirac equation. Schrödinger labeled the motion of an electron in such a state as *zitterbewegung*.

This labeling suggests an electron fluctuating between positive and negative energy while its spatial position also oscillates rapidly. As addressed in Section II A, the amplitude of this spatial oscillation is of the order of the Compton wavelength ($\sim\hbar/m_e c$), and the frequency of this fluctuation is the *zitterbewegung* frequency of the Dirac electron, of the order of $2\, m_e c^2/\hbar \approx 1.5 \times 10^{21}\ \mathrm{s}^{-1}$.

Wilson then considers the electron as having or being an oscillation of charge, defined as a center of charge or CoC coordinate operator that oscillates or spins extremely fast—a set of three independent one dimensional harmonic oscillators with zitter frequency of $1.5 \times 10^{21}\ \mathrm{s}^{-1}$—around the electron's apparent center of mass or CoM. This CoM is not to be confused with the standard point particle view of the electron, because the CoC occupies a location at a time, so it is not a charge distributed in a volume, thus avoiding the infinite self-energy feature that the point particle implies and that is dealt with by the screening mechanism of virtual particle creation and annihilation process.

He derives the electron's CoC operator from the Dirac equation (DE) such that in the rest frame it is located on a spherical shell with a radius of the order of its Compton wavelength, and in the non-rest frame, the electron's CoC operator is located on an oblate spheroid, flattened by the special relativity factor in the direction of its CoM velocity and by the speed of light c in the directions perpendicular to its CoM velocity, as seen in Fig. 11.

Wilson departs from the position operator CoC from Sakurai,[100] hereafter, in Heisenberg representation,

$$\begin{aligned} x_k(t) &= x_k(0) + c^2 p_k H_D^{-1} t \\ &\quad + (ic\hbar/2)\left(\alpha_k(0) - c p_k H_D^{-1}\right) H_D^{-1} e^{-2iH_D t/\hbar}, \end{aligned}$$

where $H_D^{-1} = H_D/E^2(p)$ and H_D is the Dirac Hamiltonian, being $k = x, y, z$ the index for coordinates.

The first two terms in $x_k(t)$ correspond to the mass coordinate $x_k^{CoM}(t)$,

$$x_k^{CoM}(t) = x_k(0) + c^2 p_k H_D^{-1} t.$$

This mass coordinate (like that of Ref. 101) contains the term $c^2 p_k H_D^{-1} t$ denoting the classical position of the mass and represents the classical displacement velocity concerning the electron center of mass, while the remaining quantum term where $\hbar$ appears explicitly oscillates rapidly with time according to $e^{-2iH_D t/\hbar}$ (the zitter term). The part of this term that contains α goes to zero in the non-relativistic limit as c $\to \infty$. Thus, the *zitterbewegung* of Dirac electron is considered a purely relativistic quantum effect, contrary to what Hestenes and others in this review claim.

In Wilson's approach, the CoC coordinates in the Schrödinger representation, where operators are time-independent, is given by

$$\begin{aligned} x_k^{CoC}(t=0) &= x_k^{CoM}(t=0) \\ &\quad + (ic\hbar/2)\left(\alpha_k - c p_k H_D^{-1}\right) H_D^{-1}. \end{aligned}$$

For an electron's CoM moving in the z direction, $\boldsymbol{p}^{CoM} = p_3 \hat{k}$, we have

$$x_3^{CoC}(p_3) - x_3^{CoM}(p_3) = \left(\frac{\hbar m_e c^3}{2E^2}\right)\Upsilon_3,$$

where Υ_k are the standard DE (4x4) complex matrices. $x_3^{CoC}(p_3)$ is the electron's intrinsic coordinate in the direction of its center of mass motion.

The magnitude of this difference contracts by the square of the SR factor Υ in the direction of motion,

$$\left|x_3^{CoC}(p_3) - x_3^{CoM}(p_3)\right| = \frac{\hbar m_e c^3}{2E^2} = \hbar/2m_e c\Upsilon^2,$$

being Υ the Lorentz factor

$$\Upsilon = \frac{1}{\sqrt{1-\left(\frac{v_3}{c}\right)^2}} = \frac{1}{\sqrt{1-(\beta)^2}},$$

where $\beta = v^{CoM}/c$ is also called by Wilson as the eccentricity.

For the other two coordinates $k=1,2$ which are the perpendicular directions to the electron's motion, the DEM's intrinsic coordinates of its CoC are contracted by $1/\Upsilon$, hence, the overall shape of the contraction is ellipsoidal as depicted in Fig. 11, with the minor axis placed in the direction of the electron's CoM motion.

The case $k=1$ is presented below:

$$x_1^{CoC}(p_3) - x_1^{CoM}(p_3) = c\hbar/2E^2\left[p_3 c\sum\nolimits_2 + \Upsilon_1 m_e c^2\right],$$

$$\left|x_1^{CoC}(p_3) - x_1^{CoM}(p_3)\right| = \frac{\hbar}{2m_e c\Upsilon}.$$

The additional term $(c\hbar/2)\ (p_3c/E^2)\sum_2$is related to the spin $\left[(\hbar/2)\ \sum_2\right]$ in the other perpendicular direction. With further developments, the spin components are found naturally in this formalism, implying that this motion is a real charge oscillation, and not just an intrinsic property of the Dirac electron.

For the electron in its rest frame, $\boldsymbol{p}^{CoM} = \boldsymbol{p} = 0$, so that

$$x_3^{CoC}(p=0) - x_3^{CoM}(p=0) = \left(\frac{\hbar}{2m_e c}\right)\Upsilon_3.$$

Wilson's CoC intrinsic coordinates oscillation around the center of mass CoM is given by the difference between both coordinate operators, denoted as $\Delta X_k^{CoC}(p_k)$,

$$\begin{aligned}\Delta X_k^{CoC}(p_k) &= x_1^{CoC}(p_3) - x_1^{CoM}(p_3)\\ &= \left(c\hbar/2E^2\right)\left[\left(\sum xpc\right)_k + \Upsilon_k m_e c^2\right],\end{aligned}$$

and it shows a harmonic oscillator nature since its derivative with respect to time gives

$$\Delta \dot{X}_k^{CoC}(p_k) = c\alpha_k,$$
$$\Delta \ddot{X}_k CoC(p_k) = -\left(\omega_0^2\right)\Delta X_k^{CoC}(p_k),$$

where $\omega_0 = 2m_e c^2/\hbar$ is the zitter frequency. The DEM model predicts that the electron's CoC coordinate operators $\Delta X_k^{CoC}(p_k)$ form a set of three independent one dimensional harmonic oscillators for any CoM momentum **p**, that oscillate at ω_0.

His model predicts that the CoC coordinate operators have eight discrete intrinsic coordinate eigenvalues $\pm\hbar/2E(p)\Upsilon$ in the two perpendicular to the CoM motion and $\pm\hbar/2E(p)\Upsilon^2$ in the CoM direction of motion. In the electron rest frame, $\Delta X_k^{CoC}(p_k=0) = \hbar/2m_e c$ for $k=1$, 2, and 3.

DEM also predicts a restoring force or spring constant of $K = 4m_e E^2(p)/\hbar^2$ and the discrete levels of the electron deduced from the electron coordinate harmonic oscillator, giving $E_n = \left(n+\frac{1}{2}\right)\hbar\omega_0$ or equivalently $E_n = (2n+1)$

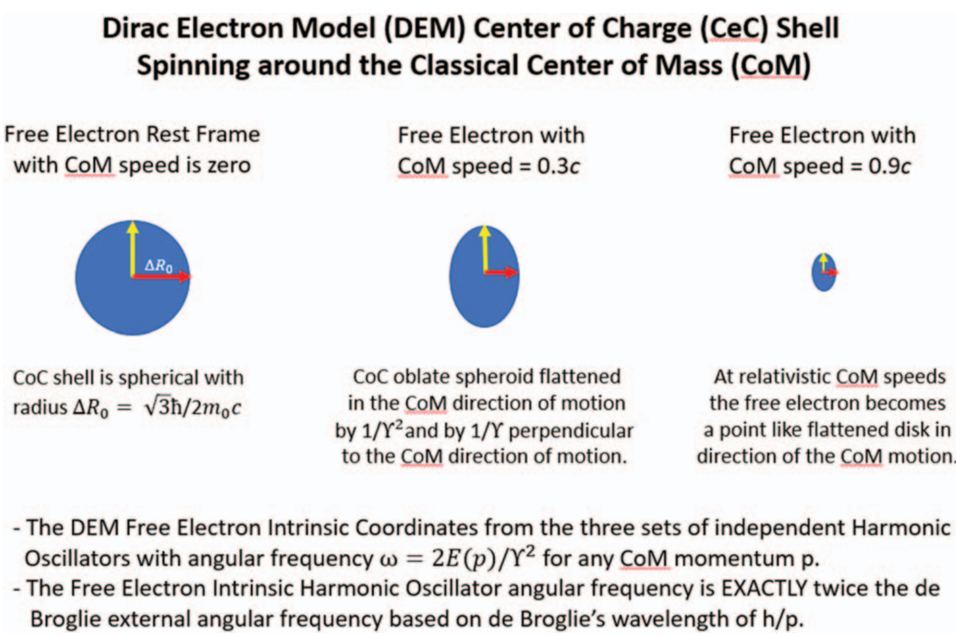

FIG. 11. Reproduced from Wilson[16] depicting the change of shape of the CoC shell as a function of the CoM speed in his DEM. The CoM direction of motion is represented by the red arrow. Image from Wilson,[16] reprinted with permission from Phys. Essays **29**, 402 (2016). Copyright 2016 Physics Essays Publication.

$E(p)$, where the ground state is the free electron's normal state.

This interesting result implies that the electron in its rest frame ($\boldsymbol{p} = 0$) needs a photon of energy $2mc^2$ to reach the first excited state, and this is the same amount of energy that an electron needs to overcome the negative energy zone in Dirac's hole theory, where a positive energy electron and a positron (hole) in the filled sea of negative energy are produced.

To complete the electron's space-time four vector internal structure, Wilson adds the time component in the rest frame as

$$\Delta X_4^{CoC} = ic\Delta T = (\hbar/2m_e c)\beta,$$

being $\beta = v^{CoM}/c$ the eccentricity. This intrinsic time operator IT implies that the electron "blinks" with adjacent time ticks of $\hbar/2m_e c^2 = 6.4 \times 10^{-22}$s apart, in comparison to the continuous nature of the time parameter associated with its classical CoM motion. This non-classical feature of IT hints into the non-classical behaviors of the electron. The discreteness of the electron CoC in space and time is caused by the discrete velocity operator $c\boldsymbol{\alpha}$ and provides a physical picture of rapid virtual electron/positron pair production, where the virtual positron annihilates the existing electron CoC. This process happens so rapidly that it "seems" continuous in the far field. From Wilson's calculations, the Dirac equation predicts that this electron's CoC's rapidly oscillating motion creates a current over time that is the source of the electron's spin and magnetic moment.

This interesting feature connects to Davis calculations showing that the electron charge is jumping back and forth across a separation of the order of $\hbar/m_e c$ at the speed c ($dx/dt = c\alpha$). As Davis remarks, the continuously varying position operator x(t) is not Hermitian, so it does not represent a continuous displacement of the charge, and hence there is no charge acceleration. When working with the Hermitian operator $x_k^\dagger x_k$ we find the magnitude of the distance, such that the electron charge apparently covers this distance at the speed of light. So, according to his model, the charge disappears at one point in space-time and reappears at another point which has a time ordered light-like separation from the first. Since the zitter electron is separated from its "twin" by a light-like interval, we could picture "one" electron located at r at time t experiencing the potential of the "other" electron at r' at time t'.

In addition, in Wilson's view the chaotic fluctuations of the electron's CoC with time creates the electron's EM field propagated throughout space; its correlated phase will be important in the physical mechanism proposed by DEM for quantum entanglement. The chaotic nature of such fluctuation is to be reconsidered, as we will comment on later in the discussion section.

This all means that the fast production of virtual electron/positron pairs and annihilation fluctuations producing the internal space and time coordinates (ISaTCOs) result from this internal electron harmonic oscillator restoring force K which origin remains unknown.

Wilson's results utilizing the Dirac Eq. with no further assumptions beyond the fact that the Dirac Eq. provides a physical description of the free electron, brings to the conclusion that the electron is not a static point particle. It only appears to be a point particle for long averaging times ($\gg 10^{-21}$ s) and large distances ($\gg \hbar/m_e c$) from the electron's CoC shell. Therefore, the electron's instantaneous CoC position is a single discrete point that is in a rapid oscillation about its CoM with the harmonic oscillator frequency of $\omega_0 = 2m_e c^2/\hbar$. For averaging times large compared to its zitter period of 1.5×10^{-21} s, the electron appears as a uniform continuous spinning CoC shell of charge e. The electron appears over time as a current caused by the "spinning" charge e on the CoC shell, in the same spirit of Vasallo's view.

From DEM straight forward calculations, one can estimate the intrinsic magnetic moment, spin and self-energy of the electron, and the Darwin term, using the DEM spherical shell radius $\Delta R_0{}^{CoC} = \sqrt{3}\hbar/2m_e c$ and the frequency of oscillation obtained by Wilson.[15] The electron self-mass in the rest frame estimated by Wilson is less than 0.7% of the observed electron mass.

Since DEM is based on the same electron velocity operator $c\boldsymbol{\alpha}$ of Dirac's equation, both formalisms provide the same predictions for critical measurements as the ones mentioned in the paragraph above. This implies that the Dirac equation is accurate, and it depicts a free electron that is really oscillating, it is a physical rotation because the electron has an inner structure. The harmonically oscillating CoC coordinate operator spins at zitter frequency around its CoM coordinate, creates a current over time which is responsible for the spin and magnetic moment, its finite self-energy, and other features that emerge from his DEM model.

Direct observations of the electron's zitter is extremely difficult because any direct observation of the electron's internal structure is impossible as it produces electron/positron pairs, destroying the free particle DEM nature. Keeping this in mind, Wilson proposes a Compton scattering experiment involving a single almost free electron undergoing elastic scattering interactions with incoming photons.[16,17] To test the DEM, he suggests that now that a single electron has been isolated[20] one can currently address the problem of Compton scattering from a single electron using both the static point electron model and the DEM electron with its spatially extended CoC oblate spheroidal shell[16,17] and compare their results.

Wilson also points out that the idea that zitter is not incorporated into QED is a misconception since the QED "finite radius" described by his DEM, shows that QED already is taking this feature into account. QED implicitly incorporates the same DEM electron properties since the same four current operator c($\boldsymbol{\alpha}$,I) is used, but QED's perturbation theory screens the physical description of the electron. Wilson coins his model QED-P (for physical), so that this is made clear. In his QED-P the electron propagator is cut off naturally via the internal coordinates structure, and the electron self-energy is finite, with no need for renormalization. This last makes his approach very robust.

Wilson's internal space and time charge operators (ISaTCOs) together create a current that produces spin and magnetic moment operators, and the electron no longer has "intrinsic" properties since the ISaTCO kinematics define these properties quantitatively. As he remarks, without the four vector current c(**α**,I) QED's calculations would be inaccurate and the electron's ISaTCO vacuum fluctuations on hydrogen energy levels would never be observed. The electron velocity operator c**α** defines its CoC vacuum fluctuations and has eigenvalues $\pm c$. This implies that negative energies and virtual electron/positron pair production occur within the electron internal structure defined by the ISaTCO. Therefore, QED and QED-P are complementary theories based on the same CoC current operator c(**α**,I), but QED-P predicts a possible physical reason for electron/positron entanglement, a feature that remains unexplained in the standard QED interpretation.

Based on a logical extrapolation of the DEM theory, in his paper[19] Wilson suggests a natural explanation for the quantum entanglement of an electron-positron pair created in vacuum and moving in opposite directions from each other. Since the electron is a stable fluctuation in the vacuum of a rapidly rotating CoC about its classical CoM, the phase component would have a physical meaning, as Hestenes also remarks. Wilson's calculations show that the two-four component positive energy solutions to DE represent the spin up and down states of the electron such that the electron is in a superposition of both spins simultaneously before measurement, and in a single spin upon measurement.

In the current understanding of quantum mechanics, particles such as the electron are weird objects in which their phase, just as their "up or down" spin is considered an intrinsic property without any physical meaning because of the imaginary nature of the wave function, where only its amplitude squared can be associated with an observable. But in the DEM theory if one reverses the phase, the Dirac equation wave function solutions describe negative energy electrons whose phase moves backward in time; these being the positrons in the DEM theory.

With this image in mind, entanglement can be interpreted physically from the DEM model as follows. When a virtual particle pair is created, it is entangled, meaning by this that their phases are locked 180° out of phase (as long as their spin states remain unchanged). During measurement, the electron spin is being altered, and it delivers a phase message through the vacuum at the speed of c^2/v^{CoM}, the electron's phase velocity. This is the speed of light divided by the eccentricity β. For $\beta = 0.25$, the phase velocity is four times the speed of light and this is the speed the electron's phase information travels. Advances in technology may allow for this prediction concerning entanglement to be experimentally verified in the future.

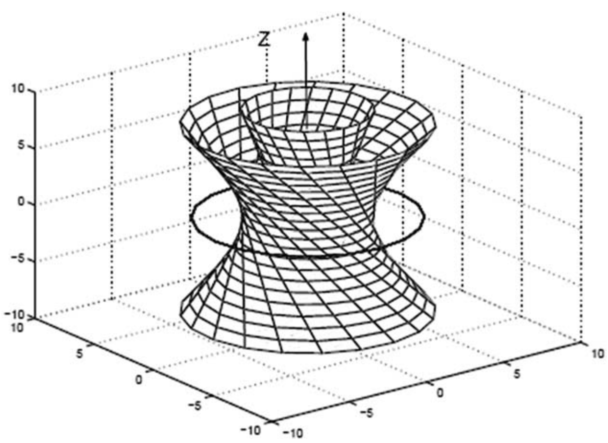

FIG. 12. The light like Kerr metric determines space-dragging caused by mass and spin. Two sheets of Kerr metric correspond to r < 0, and r > 0. Figure from Ref. 61, published under CC-BY license by IOP Publishing Ltd.

Given these remarkable, straightforward results obtained by Wilson, which agree with QED and the Standard Model for the anomalous magnetic moment of the electron, and Lamb shift, it is strange the little attention that such a well stablished framework has received, even more considering that the main aspects of its proposal for the internal structure of the electron can be tested with current technology.

VI. THE DIRAC–KERR–NEWMAN ELECTRON MODEL

Alexander Burinskii, from the Nuclear Safety Institute at the Russian Academy of Sciences, has addressed extensively the electron puzzle and remarks that the assumption on weakness of gravity is consequence of having underestimated the role of spin, which in elementary particles is extremely high.

As Burinskii remarks:[60] "Indeed, nobody says that gravity is weak in Cosmology where physics is determined by giant masses. Similarly, the giant spin/mass ratio of spinning particles makes influence of gravity very strong in the particle physics." Since in relativity theory, spin is inseparable from rotation, Burinskii explains that spin creates a gravitational frame-dragging or Lense-Thirring effect in Kerr geometry distorting space along with mass as depicted in Fig. 12.

Being the spin/mass ratio of the electron about 10^{22} (dimensionless units $G = c = \hbar = 1$), Burinskii explains that its influence becomes so strong that conflict with quantum theory shifts from Planck to Compton scale. The spinning Kerr-Newman (KN) solution—which has gyromagnetic ratio $g = 2$ corresponding to Dirac theory of the electron—and its structure for huge spins provides a hint for the origin of the incompatibility between gravity and quantum theory. As he explains in his work, quantum theory requires flat space in Compton zone, but as electron spin $J = \hbar/2$ exceeds mass m impact by 22 orders of magnitude, spin breaks the topology of space by creating a singular ring of Compton radius $a = \hbar/2m_e$, i.e., a discontinuity or singularity which as we know from black holes physics, is synonym of huge space-time deformation and hence, gravitational force effect like a black hole (*please note that Burinskii represents here the Compton radius as a, not* r_C). In this KN case, the black hole horizons disappear, opening the naked Kerr singular ring of the Compton radius $\sim 10^{-11}$ cm, very far from the expected point-like electron of quantum theory.[102]

Instead of treating it as a singularity, Burinskii proposes an alternative solution based on supersymmetry *bag models*. Bag models are placed between strings and solitons models, and just as solitons, they are nonperturbative solutions of the Higgs field model. In these supersymmetric bag models, the gravitational field is expelled from the Compton zone of the spinning particle, similarly to the expulsion of the electromagnetic field from a superconductor. Such expulsion of gravity and electromagnetic fields, create three zones:

(I)—flat quantum Interior,
(E)—External zone with exact KN solution,
(R)—zone of transition from (I) to (E).

The structure of a bag is determined almost unambiguously because given the giant values of spin, these demands become very restrictive. The resulting effect gives a sort of form factor or structure to the electron.

Zone (R) corresponds to $r = R = \frac{e^2}{2m}$ and the bag takes form of a disk with thickness R and radius $r_C = \sqrt{(R^2 + a^2)}$ as seen in Fig. 13.

As the symmetry breaking embedded in the Higgs model is used in nonperturbative electroweak models, it is natural to use Higgs mechanism of symmetry breaking to satisfy (I), (E), and (R). Additionally, its corresponding Lagrangian is also known as Landau-Ginzburg (LG) field model for superconducting phase transitions, together with the Nielsen-Olesen (NO) model for vortex string in superconducting media having potential V with typical form that distorts the external KN solution and places Higgs field in zone (E).

Burinskii verified through his developments that the conditions (I), (E), and (R) are satisfied by supersymmetric LG model with three Higgs-like fields that keep the Lagrangian almost the same but change considerably the form of the potential V, determining two vacuum states where $V = 0$:

(I)—internal: $r < R - \delta$ where Higgs field$|H| = \eta$,
(E)—external: $r > R + \delta$ where Higgs field$H = 0$,
which are separated by a zone of phase transition (R), $V > 0$, in correspondence with (I),(E),(R).

Bag models with potential V form "cavity in superconductor," while supersymmetric potential concentrates Higgs field in zone (I), forming supersymmetric and superconducting vacuum state inside the bag and showing that

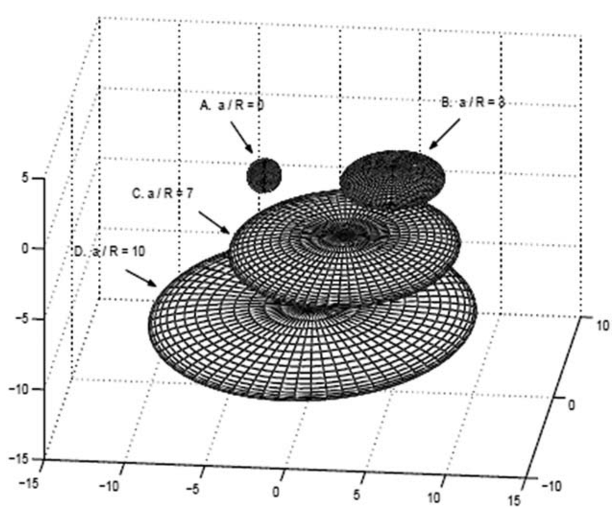

FIG. 13. Shape of disk for different $a = J/m$: (A) $a/R = 0$, (B) $a/R = 3$; (C) $a/R = 7$; and (D) $a/R = 10$. Figure from Ref. 61, published under CC-BY license by IOP Publishing Ltd.

current $J\mu = 0$ is concentrated in the surface layer superconducting disk. A remarkable consequence in the model proposed by Burinskii is the quantization of angular momentum $J = n/2,\ n = 1,\ 2,\ 3\ldots.$ which he explains happening as follows: near the boundary of the disk $r = R = e^2/2\, m_e$ where $\cos\theta = 0$, the vector-potential $A\mu$ is dragged by Kerr congruence (Fig. 12) forming a closed loop along a singular ring called a Wilson loop. The angle θ signals the direction of the dragging. The spinning gravitational field sets the shape of the bag accordingly (R), and circular string is formed on the boundary of the disk, closely to Kerr singular ring (see Fig. 13).

As remarked by Burinskii,[60] the choice of the Planck scale as the universal scale for all unifications raised the following problem: when unifying quantum theory and gravitation, the second refuses point-like structureless quantum particles as it requires extended field structure for the right side of Einstein equations $G\mu\eta = 8\pi T\mu\eta$. Here, η and μ are parameters in the metric of the corresponding Minkowski space M^4. At the same time and since gravity is a supposedly weak force, particle physics attempts to bring together the gravitational and weak force scale by explaining the weakness of gravity through gravity "leaking" into a higher-dimensional bulk. This allows superstring theory to realize scenarios for any numbers of extra dimensions. The detection of gravitational waves (GW) has been a remarkable tool for testing different theories describing gravity, such as this. The collision event of two neutron stars detected in LIGO on August 17, 2017—GW170817—allowed comparison of the electromagnetic waves accompanying the gravitational signal. The large-wavelength gravitational waves and short frequency photons experienced the same number of spacetime dimensions; no evidence was found for extra spatial dimensions, GW170817 is fully consistent with General Relativity.[103]

To summarize this section, Burinskii emphasizes weakness of gravity in particle physics as being an illusion caused by underestimating the role of spin in gravity. Relativistic rotation is inseparable from spin which for elementary particles is extremely high and exceeds mass force by 20–22 orders of magnitude. Such a huge spin generates frame-dragging that distorts space much stronger than mass, and effective scale of gravitational interaction is shifted from Planck to Compton distances. He shows that compatibility between gravity and quantum theory can be achieved without modifications of Einstein–Maxwell equations using a supersymmetric Higgs model of symmetry breaking and forming a nonperturbative super-bag solution which generates a gravity-free Compton zone necessary for consistency with quantum theory.

This theory is consistent with Higgs mechanism of the standard model while at the same time incorporating a form factor that would be compatible with the Einstein Fields equations. It also includes the strong force mass which would remove the weak force regime limitation raised by the standard Higgs mechanism—and so accounting for all mass and not just the one coming from weak interaction-. But above all, when emphasizing the crucial role of spin in the whole dynamics and unification problem, this work provides the missing link that together with the former descriptions and more particularly with the next model, contribute to the complete picture of it all.

As we will see in Section VII, an appropriate geometrical description for a volumetric unit density of space, called Planck spherical unit—PSU, together with the inclusion of a thermo-dynamical potential transfer between surface and volume of a spherical system, in terms of PSUs (which are quanta of action or angular momentum), allowed Nassim Haramein to achieve his solution to quantum gravity[104,105] by predicting the charge radius of the proton within 1σ agreement to the latest muonic and electronic measurements[106,107] and with no adjusting parameters. Using the generalized holographic solution, Val Baker *et al.* find the mass of the electron from first principle calculations.

VII. THE HOLOGRAPHIC MASS ELECTRON MODEL

The origins of the holographic principle can be traced back to David Bohm,[108] who suggested that every region contains a total "structure" enfolded within it. In this sense, he referred to the universe as a hologram, based on its analogy with optical holography.

In the same spirit, Bekenstein[109–112] later proposed that entropy in a region of space is limited by the area of its boundary. Inspired on these seminal works on black holes thermodynamics and entropy,[109–112] Nassim Haramein arrived at a quantized solution of gravity and a relation between gravitational mass and holographic mass by establishing a nondimensional fundamental ratio ϕ that represents the volume-to-surface information transfer potential of the entity (proton, electron, black holes) taken as spheres in a first order approximation.[104,105]

The key ingredient of this theory is the unit called Planck spherical unit—PSU—which is not to be confused with a unit of measure. These PSUs are the vacuum particles like the Planck particles of Daywitt's description, which are the quanta of electromagnetic oscillations composing the vacuum fluctuations. The geometrical definition of the PSU voxel quantizing space seems to provide the missing piece of the puzzle. Through this theory based on information content and exchange between surface and volume—quantized both by PSUs—not only has he obtained the most precise value for the proton charge radius,[104,105] but he has also addressed and solved the vacuum catastrophe problem.[113] Therefore, the correct unit of energy density at Planck scale was found.

Exploring further the holographic principle and the Bekenstein-Hawking maximal entropy of a black hole,[112] Haramein proposed a generalized holographic approach in terms of both surface and volume entropy of a spherical system, while the standard holographic principle only considered the surface. Another difference is that the holographic bit of information is not defined as l^2 as commonly used,[109–112] instead it's defined as an oscillating Planck spherical unit (PSU)

$$PSU = \frac{4}{3}\pi\ r_l^3,$$

where $r_l = l/2$ and l is the Planck length.

These PSUs or Planck voxels tile along the surface area of a sphere, as seen in Fig. 14.

In this approach,[104,105,113,114] the information/entropy of a spherical surface horizon is calculated in terms of spherical bits, defining a surface information/entropy η in terms of PSUs as

$$\eta = \frac{A}{\pi\ r_l^2},$$

where A is the surface area of a spherical system, and the Planck area is the equatorial disk of a PSU taken as one unit of information/entropy. In this definition, the entropy is ≈ 5 times greater than that set by the Bekenstein Bound $S = A/4l^2$.

Similarly, the information/entropy within a volume V with radius r of space is defined in terms of PSU as

$$R = \frac{V}{\frac{4}{3}\pi\ r_l^3} = \frac{r^3}{r_l^3}.$$

In Ref. 104, it was demonstrated that the holographic relationship between the transfer energy potential of the surface information and the volume information equates to the gravitational mass of the system. Therefore, any black hole of Schwarzschild radius $r_{S,}$ has a mass m_S given by

$$m_S = \frac{R}{\eta}\ m_l,$$

where m_l is the Planck mass, η is the number of PSU on the spherical surface horizon, and R is the number of PSU within the spherical volume. This was verified for Cygnus-X black hole[104] showing that one can obtain a holographic gravitational mass, equivalence to the Schwarzschild solution, in terms of a discrete granular structure of spacetime at the Planck scale.

The only radius where all the volume information is encoded on the surface, is the condition at which the holographic ratio equals one ($R = \eta$),

$$r_{S_l} = \frac{2Gm_l}{c^2} = 2l,$$

FIG. 14. (Color online) Schematic to illustrate the Planck spherical units (PSU) packed within a spherical volume. Image from Val Baker *et al.*,[114] Reprinted with permission from Phys. Essays **32**, 225 (2019). Copyright 2019. Physics Essays Publication.

where r_{S_l} is the Schwarzschild radius of a black hole with Planck mass $m = m_l$.

In this circumstances, the surface entropy and the volume entropy give $\eta_l = R_l = 64$, which results in a holographic ratio $R_l/\eta_l = 1$ yielding $m_l = (R_l/\eta_l)m_l$. This means that the volume-to-surface information transfer potential is at a balanced state of equilibrium, supporting the conjecture that "due to its ultimate stability, the Planck is the fundamental granular kernel structure of spacetime forming a crystal-like structured lattice at the very fine scale of the quantum vacuum."[114]

Now, at the opposite scale—at the level of the proton—the granular structure clustering of spacetime leads to the precise value for the mass m_p and charge radius r_p of a proton

$$m_p = 2\frac{\eta}{R}m_l = 2\Phi m_l \;\; r_p = 4l\frac{m_l}{m_p}$$
$$= 0.841\;\;236(28) \times 10^{-13}(\mathrm{cm}),$$

where ϕ is defined as a fundamental holographic ratio η/R. This value for the proton radius is within 1σ agreement with the latest muonic and electronic measurements of the charge radius of the proton[4,106] relative to a 7σ variance in the standard approach at that time.[107]

But not only did this model work for the Cygnus-X black hole and the proton; Val Baker *et al.* arrived at the electron mass and radius as well.[104] By means of this approach, the mass of the electron in terms of quantum electromagnetic vacuum fluctuations is found. As the abstract in the electron paper summarizes[114] "The solution gives a clear insight into the structure of the hydrogen Bohr atom in terms of the electron cloud and its relationship to the proton and the Planck scale vacuum fluctuations. Our electron mass solution agrees with the measured CODATA 2014 value. As a result, an elucidation of the source of the fine structure constant, the Rydberg constant and the proton-to-electron mass ratio is determined to be in terms of vacuum energy interacting at the Planck scale."

The generalized holographic approach sees the mass as emerging from the granular Planck scale structure of spacetime in terms of a surface-to-volume information transfer potential, which decreases with increasing radius. To extend this framework to the electron, authors thought the electron could be a cloud of potential energy spatially extending from the proton out to the radius where the volume encloses the electron cloud of a hydrogen Bohr atom. Therefore, in this frame the electron is no longer considered a separate system.

An additional consideration is the fact that protons are spinning. It would be reasonable to suppose that the spinning or angular speed of the proton would subject the system to special relativity and mass-dilation, which varies as a function of speed. And this speed is a function of the distance to the rotation center, i.e., to the radius.

If we evaluate the inverse of the Lorentz factor ($\Upsilon^{-1} = m_0/m$) as a probe particle is moving away from a proton rotating as close to light speed c as possible, and so decreasing its speed, and plot Υ^{-1}, we obtain the Fig. 15, where we see to the left of the plot that mass m starts at the

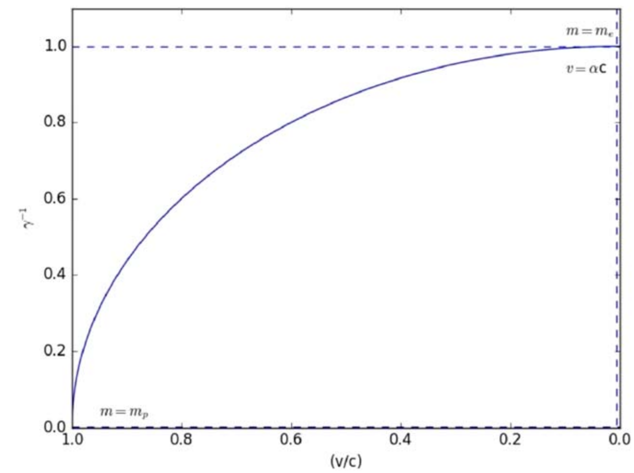

FIG. 15. (Color online) Plot of the Lorentz factor $\Upsilon^{-1} = \sqrt{1 - (v/c)^2}$ as a function of velocity for the proton to the electron.

rest mass of the proton $m = m_p = m_0$, and as we move away to the right in the horizontal axis, speed decreases (and so the ratio v/c increases), and mass m decreases too, becoming the known mass of the electron m_e ($m = m_e$) when speed has reached the value $v = \alpha\, c$, which is the expected velocity for the electron in the first orbital of the Bohr hydrogen atom ($v \approx c/137 = 2.18 \times 10^6$ m/s), being α the fine structure constant ($\alpha \approx 1/137$).

The features that we have just described about Υ^{-1} are very telling. They tell us that there seems to be a continuous transition from proton mass to electron mass, expressed through their speeds. Therefore, instead of thinking about the electron as a separate system like a particle orbiting the nuclei, the electron could instead be thought of as an inherent feature of the proton dynamics; a distribution or cloud of potential energy spatially extending from the proton out to the radius where the volume encloses the electron cloud of a hydrogen Bohr atom, since we saw from Fig. 15 that the mass of the electron m_e happens at the expected velocity for the electron in the first orbit of the Bohr model for the hydrogen atom. In Bohr's model, the Bohr radius (a_0) is a constant that gives the most probable distance between the nucleus and the electron for a hydrogen atom in its ground state.

Therefore, authors considered the holographic ratio relationship as we extend the radius of the co-oscillating Planck PSUs beyond the charge radius of the proton ($r > r_p$) being r_p the proton charge radius. It is thus reasonable to consider a velocity relationship in the holographic mass solution which becomes significant at speeds lower than c ($v < c$) which in this case would appear at $r > r_p$. Using the holographic approach, we should expect that as the radius r is larger than r_p (i.e., for all $r > r_p$) the mass m becomes smaller or decreases the mass of the proton m_p ($m < m_p$).

These considerations made the authors realize that the holographic equation they had already derived for the proton mass m_p ($m_p = 2\phi m_l$) could also be interpreted as a mass dilation since m_p is much smaller than the Planck mass m_l as if the velocity relationship was already taken into account inside this factor 2ϕ, and all they had to do in order to extend this velocity relationship as we extend the radius r beyond the proton radius r_p (and therefore the holographic surface-to-volume ratio in terms of this variable radius r), was to replace the factor 2 in the holographic solution for the proton, with a general and unknown geometric parameter β, obtaining a general equation $m_r = \beta\phi_{r,l}\, m_l$ that accounts for a decreased mass m_r as we move farther away from the proton radius r_p.

This expression for a mass "dilation" now depends on a holographic surface-to-volume ratio $\phi_{r,l}$ given in terms of PSU that tile a surface and fill in a volume (where both depend on a variable radius r) "scaled" by β, therefore, β could be related to a Lorentz factor but in terms of the holographic ratio concerning distances, instead of speeds, what evidently suggests an angular momentum velocities relationship, just as the Lorentz factor Υ which depends on the speed relationship v/c.

When the radius r reaches the Bohr radius a_0 (i.e., when $r = a_0$) and using a geometric factor $\beta = 1/(2\alpha)$ authors find a mass in precise agreement with the experimental mass of the electron. This confirms the initial hypothesis of the system proton-electron as a continuous entity. The solution for the mass of the bounded electron can thus be given as

$$m_e = \frac{1}{2\alpha}\Phi_e m_l,$$
$$\Phi_e = \frac{\eta_e}{R_e} = 4\frac{r_l}{a_0} \quad \eta_e = \frac{4\ \pi\ a_0^2}{\pi r_l^2} \quad R_e = \frac{a_0^3}{r_l^3},$$

where r_l is the Planck radius $r_l = l/2$, being l the Planck length.

With this solution for m_e they found a mass $m_e = 9.10938(30) \times 10^{-28}$ gr which is accurate within 1σ and with a precision of 10^{-5} compared to the measured CODATA 2014. As authors remark, the holographic solutions are restricted by the value of the Planck units which depend on experimental values of the gravitational constant, G. Nevertheless, the absolute value of the holographic mass solution for the electron is comparable with the experimental CODATA 2014 value, to a greater degree of accuracy $<1\sigma$ and a closer precision of 10^{-8}.[114]

This significantly accurate holographic solution clearly shows that there are differential angular velocities of the collective coherent behavior of Planck information bits (PSU) determining specific scale boundary conditions and mass-energy relationships. These specific boundary conditions are defined by the general and complete geometric factor $\beta\phi_{r,l}$ (an analogous to a Lorentz factor for the generalized holographic model) in the expression for m_r that decreases from Planck mass to smaller mass particles as we move away from the Planck radius r_l.

This holographic solution is directly related as well to the hierarchy problem, since the mass of the electron is a function of the Planck vacuum oscillators surface-to-volume holographic relationship, over a region of spacetime equivalent to the Bohr hydrogen atom (i.e., extended to a maximal radius a_0). As Val Baker *et al.* claim in their work[114] "In much the same way that the electron analogy is proposed to resolve the Higgs hierarchy problem, with the inclusion of virtual supersymmetric particles, we could also assume that the surface-to-volume holographic relationship in the Higgs region of space would solve for the mass of the Higgs, where the Higgs radius would be of the order, $r_l < r_{HIGGS} < r_p$."

This relates as well to Daywitt's succession of Compton relations $r_C\, m_e c^2 = r_l\, m_l\, c^2 = e_l^{\,2} = c\ \hbar$ that tie the electron $r_C\, m_e\, c^2$ to the Planck particles $r_l\, m_l\, c^2$ within the Planck Vacuum. Meanwhile, just as with the *zitter* effect, the current quantum understanding explains the hierarchy bare mass problem for the electron mass considering antimatter, where positron and electron pairs pop in and out of the vacuum, smearing out the charge over a greater radius such that the bare mass energy is cancelled by the electrostatic potential.

Directly applying the holographic solution, we obtain the ratio μ of the proton mass m_p to the electron mass m_e,

$$\mu = \frac{m_p}{m_e} = 4\alpha\frac{\Phi}{\Phi_e} = 4\alpha\frac{a_0}{r_p} = 1836.152(86).$$

The solution has been extended to include other radii below the Bohr radius, reaching all known elements of the periodic table and therefore defining the atomic structure and charge because of the electromagnetic fluctuation of the Planck scale. Furthermore, the atomic number Z emerges as a natural consequence of this geometric approach.

They also obtain the fine structure α and Rydberǵs constants by direct substitution

$$\alpha = \frac{\Phi_e\ h}{8\pi\ r_l m_e c} = \frac{\Phi_e \lambdabar_e}{8\pi r_l} = 7.29735(34) \times 10^{-3},$$
$$R_\infty = \frac{\alpha\Phi_e}{8\pi\ l} = 1.097373(36) \times 10^{-5}\mathrm{cm}^{-1},$$

where $\lambdabar_e$ is the reduced Compton wavelength ($\lambdabar_e = \lambda_C/2\pi$). This reduced Compton wavelength appearing in the above expression, is the Compton radius r_c mentioned all throughout this review ($\lambdabar_e = r_c$). When analyzing the expression for fine structure constant α we observe that it is a Compton wavelength to a Planck wavelength relation, and this suggests the same electron nature as expressed by the zitter oscillating terms that Schrödinger and Breit found when analyzing Dirac's solutions to the free relativistic electron; an oscillation with amplitude of magnitude of the electron Compton wavelength λ_C.

No other current theory has derived with such accuracy and from first principle calculations alone, the mass and radius of the proton and the electron. The generalized holographic solution for the electron implies as well that the atom—proton and its electron shells—could be seen as black holes. Such parallelism between Bohr atom and black holes had been suggested by Lemaitre back in 1927 through his hypothesis of the primeval atom, and very recently by Christian Corda.[115] In Corda's model a black hole is like the semi-classical model of Bohŕs hydrogen atom, where the quantum normal modes (QNM) represent the electron jumps and the absolute value of the QNMs frequencies triggered by emissions (Hawking radiation) and absorption of particles, represent the energy shells of the gravitational hydrogen atom.

To conclude this section, we would like to stress out the thermodynamic aspect to the holographic solution for the electron and its extension to all atoms, associated with the holographic ratio ϕ which is a volume-to-surface rate of information transfer. The holographic ratio ϕ is like the kinetic constant k(T) used in chemical reactions. In the case of ϕ, it could be a kinetic or reaction rate constant associated with a Gibbs free energy concerning the surface-to-volume information exchange. In this sense, ϕ represents a thermodynamic steady state calculation.

VIII. DISCUSSION

In this review, we have gathered electron models alternative to the predominant view of the electron as a structureless nondimensional point like charge with momentum and position determined by a probabilistic interpretation of the wavefunction described in terms of an electronic density cloud. The common denominator of most of these approaches is the underlying notion that the vacuum fluctuations are real mechanical features of the vacuum, which therefore acquires some level of structure and organization that give rise to particle's *zitterbewegung*, among other features. Because the connection between the zero-point vacuum fluctuations and zitter is not yet fully understood or explained in a unified framework, we divided the electron models descriptions into two groups: the electron models from Section II explain zitter in terms of a real light speed circular motion of the electron with Compton radius r_C, and, Sections III–VII address the electron zitter models in terms of zero-point energy vacuum fluctuations zpe. Such division is not absolute as there is overlapping between models and sections/subsections.

The zitter models included in Section II are: Section II A: *Schrödinger's interpretation of zitterbewegung*, Section II B *Zitterbewegung interpretation of quantum mechanics,* from David Hestenes, Section II C *The electron as a 2D harmonic oscillator*, from Jean L. Van Belle, Section II D *The ring and the helical electron model*, from Oliver Consa, Section II E *The Superluminal quantum model of the electron and positron* from Richard Gauthier, and finally, and Section II F *The zitterbewegung electron model and Occam's razor*, by Giorgio Vasallo *et al*.

The origin of zitter could probably be traced back to the vacuum fluctuations, thought to be the source of the electron inner structure, charge and mass as proposed in models of Section III. *Stochastic electrodynamics*, from Haisch *et al*., Section IV. *Dirac electron model in the Planck Vacuum Theory*, from William Daywitt, Section V. *Dirac Electron Model and QED-P* from James Wilson, Section VI. *Dirac Kerr Newman electron model*, from Alexander Burinskii, and Section VII. *The holographic mass electron model*, from Amira Val Baker *et al*.

Section V "Dirac Electron Model and QED-P" provides a framework to connect the semiclassical models and the fully relativistic QED depiction of particles, as well as zitter with zpe. Wilson's approach shows that the electron's CoC rapid harmonic oscillation about its CoM creates a charge shell and magnetic moment over time that determines the intrinsic electron properties with DE quantum position and velocity operators, where the predictions of the Standard Model such as the lamb shift and the anomalous magnetic moment of the electron, remain unchanged. Wilson shows that his DEM model is an extension of QED, that he coins QED-P (for physical) because he can show how these oscillations explain the features which would therefore be physical instead of intrinsic to the electron, and his results can be tested experimentally.[16,17]

Experimental confirmation of the electron zitter would demonstrate that there is a natural photon propagator cut off energy determined by the Dirac equation and that QED is free from ultraviolet divergences of the electron self-mass (this also happens with Haramein-Val Baker's approach that provides a natural cutoff, in terms of the energy density of the PSU given by the Planck mass in a spherical volume of radius $r_1 = l/2$, being l the Planck length). A statement from Barut complements this nicely:[32] "if the electron is not a point particle, but a massless charge performing a complicated motion around a center of mass, such a picture cannot possibly be without implications for the self-energy and renormalization problems of the electron." Barut also remarks that interesting geometrical and dynamical substructure associated with Dirac's equation cannot be revealed unless the superpositions of positive- and negative-energy states of the "one-particle system" are considered, in agreement with Wilson's work.

An interesting point addressed by Wilson is the electron self-mass in the rest frame estimated using the DEM's CoC radius gives less than 0.7% of the observed electron mass, therefore it is not infinite. Wilson also remarks that DE and his DEM are not a single particle equation, it is an odd particle model, and as he explains:

"The electron described by the Dirac Eq. and the ISaTCO (Internal Space and Time Coordinate) is never one particle. It is a three, or five, or seven, or…. 2n − 1… particle model, with a point electron located at its CoC in the presence of a virtual electron/positron pair, whose positron annihilates the original electron leaving behind the electron from the virtual pair at a very rapid rate ($\sim 1.4 \times 10^{-21}$ s). There can be any number of virtual pairs present, but there is 1/137.04 less likely to be two sets of virtual electron pairs than one, and $(1/137.04)^2$ less likely to have three sets of electron/positron pairs than one, etc. This virtual pair production and annihilation occurs as vacuum fluctuations on a very rapid time scale of $\sim 1.4 \times 10^{-21}$ s. for the electron, and approximately 207 times more rapidly for the muon. In fact, the electron is a field with phase fluctuations throughout all space created by the fluctuating CoC" (Ref. 18, p. 19).

From Wilson's perspective the electron could be seen as a steady state of a dynamical and very high energetic vacuum fluctuations mechanism in the region of space that we call "electron." As Wilson explains, the electron is not only located at its Bohr orbit, otherwise, the electron's ISaTCO vacuum fluctuations on hydrogen energy levels would never be observed; the stability inside the volume of a proton in the S-level energy state allows for the impact of electron's CoC vacuum fluctuations to be measured.

He states:[18] "The spatially extended proton, with the electron and muon CoCs traveling at the speed of light around their CoMs inside the proton predicted by QED-P, is a bizarre picture, but this situation is absolutely confirmed by QED's very accurate estimates of the hydrogen and muon energy levels …//… There is a need for a much more accurate model of the proton's internal structure …//… If the proton radius is scaled up to 0.5 cm. (about half the size of a pea), the muon CoC would be approximately 6 m from the proton's CoM at the origin, while the electron's CoC would be over 1200 m from the proton's CoM at the origin. In both cases the extended proton is well 'within' the electron or muon CoC shell, and the proton's quarks experience significantly different EM field fluctuations from induced the electron or muon. Even though this situation is chaotic, QED calculates the electron and muon interactions with their ISaTCO vacuum fluctuations very accurately in estimating the resulting hydrogen energy levels."

This is very relevant in the frame of the generalized holographic model in which both proton and electron radius and mass are successfully explained in terms of the EM vacuum fluctuations represented by the PSUs, from first principle calculations and free from any adjusting parameters. Haramein's results clearly show that it is this surface-to-volume ratio which explains the emergence of mass in the proton, followed by Val Baker results that clearly indicate that the electron is not only at the Bohr radius. In fact, it is an extended coherent field of vacuum fluctuations that extend from the proton and up to the Bohr radius, and the same mechanism applies to explain the proton itself, as a coherent and stable condition of the collective quantum vacuum fluctuations.

The collective behavior would imply that this is not a chaotic oscillation, as we saw in Section VII for the solution of the electron and the holographic mass, in combination with Daywitt's Planck Vacuum Theory PVT results from Section IV that show a relationship of angular momentum conservation as well, implying it is a coherent oscillation in terms of angular momenta, as Vassallo also proposes.

Through the models described so far, we appreciate that zitter is considered a real oscillatory motion of the electron charge, but then a question inevitably emerges: what is the origin or source of such oscillation? The answer by default would be the quantum vacuum fluctuations, which is what Section III. *Stochastic Electrodynamics*—SED—theory explores and proposes. We recommend the following webpage where relevant information regarding this matter has been addressed http://www.calphysics.org/zpe.html. In SED, zitter is explained as an interaction of a classical particle with the zero-point field. The connection between zitter and zpe have fueled investigations for a mass generation alternative to a Higgs mechanism which can only explain the weak interaction part of the mass, accounting to 2% of the total baryonic mass.

As Haisch explains in Ref. 37, "It is important to keep in mind that the actual direct measurement of the thing we call inertial mass can only take place during acceleration or deceleration which is simply acceleration directed opposite to the existing velocity. We assume that an object always possesses something called mass even when it is not accelerating and proceed to calculate the momentum and the kinetic energy of an object moving at constant velocity with respect to us. But there can be no direct evidence that an object possesses mass unless it is being accelerated. The only way we can directly measure the momentum or the kinetic energy that we calculate is by bringing about a collision. But a collision necessarily involves deceleration. It makes for good bookkeeping to assume that an object always carries with it a thing called mass, yielding a certain momentum and kinetic energy, but this is necessarily an abstraction."

Following Sakharov,[96] Haisch proposes that gravity is not a separately existing fundamental force but rather an induced effect associated with zero-point energy fluctuations of the vacuum, just like the van der Waals and Casimir forces, and he provides a model for it. In this frame, Puthoff and others[83–92] propose a point-particle-zpe interaction model that fulfills this hypothesis. In his model, gravitational mass and its associated gravitational effects are shown to derive in a fully self-consistent way from electromagnetic-zpe-induced particle motion *Zitterbewegung*. therefore, in this form the theory constitutes an "already unified'" theory.

This brings us to the Dirac electron model in the Planck vacuum theory—PVT—from Section IV, which gives what we consider to be a very accurate attempt to discern the relation between vacuum fluctuations—zpe—and the electron. Inspired by former works as those from Puthoff and Sahkarov, Daywitt[98,99] proposes a model where the vacuum is conformed of Planck particles which conform all mass as well. He derives a succession of Compton relationships addressing the hierarchy problem in particle physics which is deeply related to the radius and mass relationships derived by Vassallo and by Val Baker *et al*.

In PVT model, the electron mass results from a massless bare charge being driven by ultra-high-frequency photons of the zero-point electromagnetic vacuum; the electron charge exhibits a small fluctuation about its center of motion, the so called *zitterbewegung* motion which would confer the mass just as SED theory, and authors in this review have proposed. The resulting massive-charge collisions with the active PV produce a cloud of electron-positron pairs around that charge, and then the massive free charge exhibits an exchange type of scattering with some of the electrons in the pairs that increase the free electron's apparent size during the process, and it is measured as such.

In Daywitt's nonrelativistic calculations for the PVT Bohr-hydrogen-atom, the Bohr quantization of the angular momentum is directly related to the electron/PV coupling force and the ratios E_n/r_n are proportional to the n-ratio from the Schwarzschild line element for Einstein field equations. This connects with the work of Burinskii who posits that the huge spin of elemental particles such as electrons, drag and deforms space-time as a black hole (BH) singularity would, fact that connects as well to Haramein's solutions to quantum gravity and the holographic mass,[104,105] where he shows that the strong force is the gravitational pull of proton size black holes. In Haramein's approach, the mass of the proton is not small, and it is simply measured as being small. This is because the relationship of the surface-to-volume ratio ($\phi = \eta/R$), when multiplied by Planck mass, produces a small energy event or mass which we call the rest mass of the proton (of the order of 10^{-24} g). However, the inverse information relationship (R/η) which is the volume-to-surface ratio, generates the holographic mass of a proton which defines its gravitational attraction; the so-called strong force, as if it were a black hole, a rotating black hole since the PSU are the quanta of angular momentum. This connects to Burinskii's model, which is based on the spin of fundamental particles, that receive an analogous treatment to rotating black holes, by means of the Kerr-Newman metric.

As remarked by Burinskii,[102] assumption on weakness of gravity results from having underestimated the role of spin, which in elementary particles is extremely high. As he explains in his work, relativistic rotation is inseparable from spin and as electron spin $J = \hbar/2$ exceeds mass m impact by 22 orders of magnitude, spin breaks the topology of space by creating a singular ring of Compton radius $r_C = \hbar/2\,m_e$, i.e., a singularity which, as we know from black holes physics, is huge curvature, i.e., gravity. Instead of treating it as a singularity, Burinskii proposes an alternative solution based on supersymmetry *bag models* which expel the gravitational field from the Compton zone of the spinning particle, just like the expulsion of the electromagnetic field from a superconductor. Through this supersymmetric Higgs model of symmetry breaking, compatibility between gravity and quantum theory is achieved without modifications of Einstein–-Maxwell equations. Additionally, his nonperturbative superbag solution is consistent with quantum theory by generating a gravity-free Compton zone.

At the time this review was being completed, Burinskii published new research[116,117] where his electron model is developed further, providing deeper arguments (such as the inclusion of the positron spin dynamics, so that the modified KN system forms a quantum electron-positron vacuum interacting with gravity) to show that the existing theories and models of elementary particles are incomplete as they neglect important effects associated with the gravitational process of the frame-dragging in the spinning gravitational spacetime. By emphasizing the crucial role of spin in the whole dynamics and unification problem, this work provides a crucial piece of the puzzle, that together with the former descriptions and more particularly with Harameińs generalized holographic model, and Val Baker's holographic electron model, complete the picture at this level. We recall that each Planck spherical unit (PSU) in Haramein's model represents a quantum of action or quantum of angular momentum, so evidently, spin is intrinsically considered in his thermodynamic steady-state calculation.

Haramein's model describes the system under consideration (PSU, proton, Universe) as a spherical object, and this first order approximation has proven to be a very good assumption. Tiling the surface and filling the volume of such a spherical system with these PSUs representing units of information, provides a surface η and volume R densities with respect to the PSUs. Since in the frame of information theory, entropy is a measure of the information content in a system, η and R are also associated with the surface and volume entropy, respectively.

With these very simple information densities named surface entropy η and volume entropy R, we obtain the fundamental holographic ratio $\phi = \eta/R$, which is a nondimensional ratio that expresses the surface-to-volume entropy, representing an information potential transfer or rate of information exchange between the volume and the surface of the spherical system. It represents a thermodynamic steady state calculation as well, reminiscent of Gibbs free energy.

The generalized holographic approach sees mass as emerging from the granular Planck scale structure of spacetime in terms of a surface-to-volume information transfer potential at all scales, from electrons and protons to cosmological black holes.

At this point, it is worthwhile mentioning the parallelism in the opposite direction—black holes as cosmological atoms, idea that has been explored by different authors. In a paper released in 2015, researchers from the Perimeter Institute for Theoretical Physics and Stanford University describe how astronomical black holes may bind surrounding particles to form a gravitational atom analogous to hydrogen, with the black hole acting as the nucleus and the surrounding particles forming a state like the electron cloud.[118]

Following a similar path, in 2018 Corda,[115] developed a model where black hole quasi-normal modes (QNMs) are interpreted as quantum levels in a semiclassical model of the structure of a Bohr hydrogen atom in which QNMs represent the "electron" which jumps from a level to another one, and where the absolute values of the QNMs frequencies are triggered by emission (i.e., Hawking radiation) and absorption of particles, represent the energy "shells" of what Corda refers to as the "gravitational hydrogen atom."

The original idea of a gravitational atom is based on principles described by Roger Penrose in the 1960s, where he showed that energy and angular momentum can be extracted from the surrounding region of black holes, proposing a mechanism by which particles could bind to the ergosphere of the black hole, through a process referred to as superradiance.[119]

Recently, it has been shown that superradiant instabilities may create clouds of ultralight bosons around rotating black holes, forming the so-called *gravitational atoms*.[120] Authors show that a binary companion induces resonant transitions between bound states of these bosonic clouds, and the interaction with the companion also triggers transitions from bound to unbound states of the cloud. This last process is referred to by the authors as "ionization," analogous to the one happening in atomic physics. And the analogy between black holes and atoms can be pushed even further; by analysis of the resulting gravitational wave signature, a sort of spectroscopic signature of the gravitational atom may be obtained.[120]

The top-to-bottom approach can logically be explored in the opposite direction, bottom-to-top, where atoms could be considered as quantum black holes. In his paper *The Schwarzschild Proton* Haramein describes "gravitational atoms" in which protons are black holes,[104,105] considering the strong binding force as a gravitational interaction, and showing that there is a screening effect on the strong gravitational force of the quantum black holes (protons), such that it follows a Yukawa potential. In this view, the electron orbit or cloud represents the ergosphere of the black hole proton.[114] Considering black holes as astronomical atoms, and atoms as tiny black holes, could provide the fundamental physical processes and dynamics to unify all scales, from quantum up to the universe.

Using geometric and thermodynamics considerations alone, Haramein's model can predict the mass of Cygnus-X Black hole and the most recent proton charge radius, with no adjusting parameters. Inspired on this model, Val Baker *et al.* obtained a model for the electron mass and radius, where instead of thinking about the electron as a separate system, it could be thought of as a cloud of potential energy spatially extending from the proton out to the radius where the volume encloses the electron cloud of a hydrogen Bohr atom. Their solution clearly proves that the differential angular velocities of the collective coherent behavior of Planck information bits (PSU) establish specific scale boundary conditions and mass-energy relationships that we call proton, electron, etc. The hierarchy bare mass problem was resolved by considering Planck vacuum oscillators acting coherently extending over a region of space equivalent to the Bohr hydrogen atom, just as Daywitt's succession of Compton relations concerning the hierarchy problem. Val Baker's solution for the electron extends the holographic mass solution to the hydrogen Bohr atom and for the ground states of all known elements, defining the atomic structure and charge as consequence of the electromagnetic fluctuations of the Planck scale. The atomic number Z emerged naturally from this geometric approach.

As was mentioned in the introduction, history has shown that the understanding of matter translates directly into the understanding of how to extract energy from it. Although considered unlikely by most physicists some years ago, the zero-point energy may become a source of usable energy, as the impossible drive and other recent devices have suggested.[121] It was proved recently that thermal motion in graphene could produce work.[122] SED proposes a way by which energy could be extracted. Based upon this knowledge, Haisch *et al.* have issued patents[123] and have conducted experiments at the University of Colorado.[124,125]

Daywitt also explores the idea of gravity control and vacuum propulsion from a fundamental physics point of view, making use of the Planck vacuum (PV) model of the vacuum state.[126] Haramein has issued various patents,[127] including his magnetohydrodynamic simulator.

As a final comment, we would like to stress out that the issue about whether it is charge and not mass what is oscillating at *zitter* frequency can only be solved unambiguously when the relation between mass and charge, that requires unification of gravity and electromagnetism, is achieved.

This implies the realization of quantum gravity. If a substantive medium is involved in such unification, this could have strong consequences on the concepts embedded within Einstein's special relativity and how they will be affected by it (inertial frame, acceleration, Lorentz transformations, inertial mass, time dilation, length contraction, etc.). Hopefully, this all could bring extraordinary changes regarding our technology and our understanding of the physical world.

IX. CONCLUSIONS

In this work, we have explored electron models that go beyond the depiction of the electron as a structureless nondimensional point like charge with momentum and position determined by a probabilistic interpretation of the wavefunction described in terms of electronic density cloud. These models share features in common that provide useful insights concerning the nature of the electron; for instance, they all consider *zitterbewegung* to be a real oscillatory motion of the electron. We compare these models, ending with the Holographic Electron Model from Val Baker *et al*., where all former approaches seem to converge. This last offers a derivation for the electron mass from first principles, obtaining a value which agrees with the latest CODATA value for the electron mass. This model is based on Haramein's holographic ratio ϕ, solution that accurately predicted the mass and radius of the proton in 2012,[104] resulting in a radius 4% smaller than the Standard Model at the time. This prediction that does not utilize adjusting parameters, was later confirmed at the Paul Scherrer Institute utilizing muons in a proton accelerator. Further experiments utilizing electrons confirmed the radius in 2017/2018. The value of the proton RMS charge radius has since been validated by the adjusted 2018 CODATA value,[128] which is the standard for all fundamental physical constants.

The non-dimensional ratio ϕ represents an information transfer potential ratio, in the same spirit of a Gibbs free energy. We stress the importance of this result, as dimensionless factors do not depend on the units of measure and the ratios involved embed truly fundamental concepts. We would include π in this list; the ratio between a circumference and its diameter, to which the quantum of action—or quantum of angular momentum depicting a complete cycle–is directly related. It is not hazardous that Planck units work well for the unification problem, since Daywitt's Planck particles (PP) or more precisely depicted, Harameińs geometrical Planck spherical units—PSU, seem to be a fundamental volume and energy density unit of space, while at the same time being the quantum of action, and so everything scales in proportion to it. The vacuum fluctuations, in this frame, being quanta of angular momentum.

We would like to extend a little further on the quantum vacuum. Because Michelsen–Morley experimental attempts to detect the luminiferous ether in 1887 reported negative results,[129] vacuum was assumed devoid of mass and structure. Fields were assumed to travel freely through nothingness. The framework based on the presumption of frictionless environment and isolated systems was established as norm. Quantum mechanics later redefined the vacuum at the microscopic scale when the zero-point fluctuations and the Casimir effect were confirmed. Since quantum weirdness as entanglement and interaction with vacuum had been assumed extremely rare and occurring only at microscale under specific experimental conditions, the isolated systems approach has prevailed. This road has brought us to the apparently dead end of dark mass and dark energy, meaning that at least 95% of our reality is accounted for by features that have not yet been detected. Additionally, the unification controversy keeps growing from both material and vacuum perspective, being the vacuum catastrophe problem the wildest evidence of a humongous misunderstanding. All our technology is based on isolated systems physics, and under this scheme, any higher order organization framework is impossible. On the other hand, it has been acknowledged that entanglement happens more often than previously thought; it has even been detected at macroscopic scale and at room temperature.[130]

Therefore, and considering that: (a) vacuum is assumed structureless and maybe it is not, as Wilczek has thoroughly claimed,[3] (b) volume in matter is more than 99.9999999…% vacuum, but we still work with mass and vacuum as if they were different entities (unification starts by unifying mass and vacuum?), and finally (c) the vacuum catastrophe problem remains a problem in leading theories, it then would not be outrageous to state that these three points are deeply connected.

Models or ideas for unification of mass to vacuum fluctuations are proposed by some authors studied in this work, concluding with the generalized holographic approach developed in Section VII, which has given very satisfactory results that are within 1σ uncertainty of their experimental values[104,105,113,114] with no adjusting parameters.

Einstein's $E = mc^2$ equation unifies mass and energy, though the origin of both energy and mass is not explained. The equation relates two unknowns, m and E, and the reason why the ratio between energy and mass is constant and denotes the maximum speed light can travel in a vacuum which in principle would have little to do with either m or E if they have not been unified to vacuum yet, remains unsolved. The quantum of action or spin at the Planck scale, the very heart of the vacuum fluctuation, seems to be the key, and all phenomena is unified through a single entity: the Planck particle or Planck spherical unit, PSU. The redefinition of the SI units in terms of a most accurate Planck constant, is then a fundamental aspect of this inquiry.

The most evident consequence of having assumed a frictionless inert vacuum at the macroscopic scale, combined with discarding a scaling of reference frames, regards the physics and technology to travel through space. Taking inspiration in the following example between a hammer-nail system, or a screw-screwdriver system, our physics uses the first case: the main translational force is the propulsion explosion and successive combustions (like hitting the nail with a hammer). The initial impulsion must be huge to overcome the gravitational field potential barrier. By using a screwdriver to drive the screw, the screw "grabs" into the "vacuum/gravity/wall" by friction with the environment through the spin of the Planck vacuum structure, by means of a density gradient through scales. Vacuum could act as the "propulsion engine" and the force comes from its torque since the Planck are quantum of angular momentum, and this is experimentally validated by the Casimir torque.[94] If we observe the universe workings, everything spins; atoms, planets, suns, galaxies … where does the energy/force to sustain these movements for so long periods of time, comes from?

We also stress the remarkable results from Haisch *et al.*,[125] where Newton's equation of motion $F = ma$ could be derived from Maxwell's laws of electrodynamics applied to the zpe field. The implication that inertia is not innate to matter and instead is an electromagnetically derived force (essentially a quantum-vacuum derived force as addressed by Puthoff's, Daywitt's, and Haramein's models) represents a breakthrough since the manipulation of electromagnetic fields is the basis of most modern technology. A paper from Puthoff[134] explains the possibility of having space propulsion based on vacuum engineering.

The extensive research on quantized inertia (QI) by McCulloch and collaborators[121]—see also publications in Ref. 131—from 2007 to date, have been tested and give a small thrust without propellant.[132,133] The company IVO Ltd, founded in 2017, has built its Quantum Drive based on principles from McCulloch's QI theory, affirming that they generate thrust in laboratory tests. IVO announced its pure electric thruster for satellites, the IVO Quantum Drive, which will be aboard SpaceX Transporter 8, providing propulsion for the satellites.

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